Open Journal of Forestry, 2024, 14, 233-280
https://www.scirp.org/journal/ojf
ISSN Online: 2163-0437
ISSN Print: 2163-0429

From Finnish Assortment Pricing to Market Economy Using Prices for Sawn Wood and Chips in Reference Bucking

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How to cite this paper: Lappi, J. (2024). From Finnish Assortment Pricing to Market Economy Using Prices for Sawn Wood and Chips in Reference Bucking. *Open Journal of Forestry, 14,* 233-280.
https://doi.org/10.4236/ojf.2024.143015

Received: May 14, 2024
Accepted: June 21, 2024
Published: June 24, 2024

Abstract

Dominant Finnish assortment pricing gives prices for sawlog and pulp wood volumes. Buyers buck stems to sawlogs using secret price matrices. Agreed dimensions allow wide range of sawlog volumes. Forest owners cannot objectively compare biddings: timber trade is a lottery game. Bucking is analyzed in terms of sawlog, pulp wood, log cylinder, sawn wood, value-weighted sawn wood, and chips. Sawn wood and its value are computed from top diameter of the sawlog. Profit maximization requires buyers to buck logs producing smaller than maximal value, causing dead weight loss. Nominal assortment prices have unpredictable relation to effective stumpage price. Assortment pricing does not meet requirements of market economy. If sawmills linked to pulp mills buck smaller sawlog percentages than independent sawmills, as generally believed, they use higher price for chips in their own harvests than they pay for independent sawmills, indicating imperfect competition for chips. Sawn wood potential pricing is suggested which gives prices for sawn wood and chips coming both from sawlogs and pulp wood in reference bucking which maximizes sawn wood for given minimum and maximum log length and minimum top diameter. Simple algorithm generates feasible bucking schedules from which optimum can be selected using any objective. Pricing produces unit price for all commercial wood utilizing ratio of theoretical sawn wood and commercial volume in stand. Unit price can be compared to stem pricing and could be compared to assortment pricing if assortment pricing would produce predictable sawlog percentages. Sawn wood potential pricing is concrete, transparent, easy to compute, considers stem size and tapering, reduces trading cost and is less risky to buyers than stem pricing. It meets requirements of market economy. Readers can repeat computations using open-source software Jlp22.

Keywords

Sawmill, Pulp Mill, Jlp22, Dead Weight Loss, Stem Price

1. Introduction

Harvesting technology (Uusitalo, 2010) and sawmill technology (Varis, 2018) set the framework where the timber trade and optimization of bucking and sawing operate. Timber trade uses timber prices to get timber from forest to sawmills and pulp mills. A good analysis and overview of different pricing methods in Finland is given in Finnish by Hekkala (2023). His thesis and references in it (e.g. Malinen & Palander, 2004; Malinen et al., 2010; Malinen et al., 2015) describe the properties and problems in different pricing methods. I analyze quantitatively well-known properties of the assortment pricing and suggest a new pricing method.

1.1. Assortment Pricing

In market economy, prices have two duties:

1) Prices help sellers to find the best buyer and buyers to find best sellers in current deals.

2) Prices transmit information between supply and demand so that sellers and buyers can optimize their future operations.

In the dominant Finnish assortment pricing, a buyer pays different price for sawlog and pulp wood volumes measured from the logs cut by a harvester. Different buyers buck stems into different dimensions and agreed dimensions of each buyer allow a wide range of sawlog volumes. The bucking algorithm in the harvester uses a secret price matrix given by the buyer. The effective stumpage price is secret when the deal is made, and it has unpredictable relation to the nominal sawlog and pulp prices. In the assortment pricing, forest owners cannot objectively compare different biddings before or after the deal even if the owner would know each stem in the stand precisely.

In assortment pricing, the relation between the effective stumpage price and the nominal sawlog and pulp wood prices is weird. Prices do not properly transmit information between supply and demand. Time horizon is long both in building sawmills and pulp mills and in growing and harvesting forests. Assortment pricing does not support long term development of forestry and forest industries. I try to shed light on viewpoints which have kept the assortment pricing alive decades after the harvesters made it obsolete from the viewpoint of market economy.

In standard trade, the seller gets the total price computed by multiplying unit price with the number of units. The unit price is agreed. The number of units can be measured from the commodity at time when the buyer gets the control of it. It may be possible that the measurements are done after the deal. In standard trade, the seller cannot influence, and has no reason to influence, the handling of the commodity after the buyer gets control of it. In the assortment pricing, buyers decide sawlog and pulp wood proportions after the deal. They can influence only slightly the total commercial volume. There are strict regulations how stem diameters and volumes are measured, and measurements calibrated but no legal rules for the bucking.

When buyers buck stems, profit maximization requires them to maximize the difference between the total value of the obtained logs (sawlogs and pulp logs) and the stumpage price. If the stumpage price does not depend on the actual bucking, in 'free bucking', profit maximization requires buyers to buck as valuable logs as possible. If the stumpage price depends on the actual bucking, the profit is maximized with logs having smaller value than obtained in free bucking. Such inefficiencies are typical when rules of market economy are not followed. They are called deadweight loss (economic inefficiency, allocative inefficiency, excess burden, or 31 other synonyms).

Some forest owners, surprised of the small sawlog volumes, have accused buyers of transporting 'valuable sawlog part' to pulp mills. Some journalists and spokesmen of forest owners have supported them. However, there is no 'valuable sawlog part' in a standing tree. A sawlog is valuable only if the sawmill can make valuable goods of it. From buyer's point of view, the 'valuable sawlog transported to pulp mill' confuses 'valuable' and 'expensive'. Potential sawlog volume transported to a sawmill is always expensive, but it may, or may not be valuable.

In market economy, there is no fair price, just a market price. A forest owner has an objective reason to be dissatisfied after seeing the obtained sawlog percentage, if the owner thinks that another buyer offering seemingly lower nominal prices had bucked a higher effective stumpage price. In the assortment pricing it is not possible to know, even after the harvest, whether the best buyer was selected. The pricing system in which forest owners cannot objectively compare biddings is to be blamed, not a particular buyer taking advantage of the loopholes of the system. Law requires joint-stock companies to make profit to the shareholders. Thus, law requires companies to mislead forest owners in the assortment pricing whenever it is profitable.

The discussion of assortment pricing has concentrated on the sawlog volume. The assortment pricing implies a permanent conflict of interest between the forest owner and the buyer. The forest owner wants to get large sawlog volume. The buyer is interested to get valuable products with minimal stumpage price. At potential cutting lengths, the volume and value of sawn wood are almost independent of the sawlog volume. The bucking cannot be understood without analyzing relation between sawlog volume, sawn wood volume and value of sawn wood. This paper provides such an analysis. A key concept is the log cylinder. If log length increases, the log volume and the length of the log cylinder increase but the diameter of the log cylinder decreases leading to either increasing, constant or decreasing volume of log cylinder. The interaction of log length, top diameter and log cylinder volume sets up the framework for the optimization of the bucking and for the inevitable conflict between the forest owner and the buyer in the assortment pricing. The top diameter of a log determines further the relation between the volume of log cylinder and the volume and value of sawn wood.

The part of sawlog which does not go into sawn wood, produces chips and bark. It is comparable to the pulp wood which produces directly chips and bark. The analysis leads to a suggestion of a pricing method, sawn wood potential pricing, where the price is based on the theoretical amount of the sawn wood and chips which come both from sawlogs and pulp logs in a reference bucking. In this pricing, a forest owner can objectively compare biddings, and the price considers the buyer's interest in the sawn wood and chips and thus sends information to the supply as prices do in market economy.

In stem pricing, one price is given for all commercial wood. A forest owner can compare different biddings in the current deal, but stem pricing does not give information to the supply-side where and when to make next harvest. Both in assortment pricing and stem pricing, commercial wood is defined to be up to given minimum diameter. Tops are often cut at a larger diameter. In the suggested sawn wood potential pricing, price is also given up to the agreed top diameter. If the last cut is done earlier, the volume of potential pulp wood can be predicted using simple equations presented. In the sawn wood potential pricing, reference bucking of sawlogs is done for healthy and undamaged parts of stems. Detection of damaged parts is the only way how the pricing depends on the realized bucking.

Buyers of timber are here classified into independent sawmills and into sawmills which are part of companies which have both sawmills and pulp mills. The latter are called sawmills with pulp. Kallio (2001) showed that competition at the chip market was not perfect due to oligopoly of chip buyers, companies owning pulp mills. If there are only few possible buyers within a reasonable distance, buyers have strong pricing power.

This paper gives indirect evidence that competition at the chip market is not yet perfect, even if energy plants buying chips have changed the market. It is a common view that sawmills with pulp buck considerably smaller sawlog percentages than independent sawmills. If sawmills with pulp use the same price for chips in their own harvests as they pay for independent sawmills, they would buck approximately the same sawlog percentages as independent sawmills.

Many forest owners are convinced that sawmills with pulp tend to buck small sawlog percentages because they want to get more pulp wood which they need in their pulp mills. When the price of chips increases, the main reason for bucking more pulp wood is initially to transfer chips from the expensive sawlogs into cheap pulpwood with small sacrifice in sawn wood and consequently with small increase of total volume of chips. The total amount of chips starts to increase considerably with very high chip prices.

1.2. Objectives of the Paper

The paper has six objectives:

1) To analyze the relations of amount of sawn wood, value of sawn wood, sawlog volume and volumes of chips coming both from sawlogs and pulp wood and their implications to the bucking optimization and pricing.

2) To demonstrate that in the assortment pricing the forest owners cannot compare objectively biddings as different buyers use different dimensions for sawlogs and the agreed dimensions allow a wide range of sawlog volumes.

3) To demonstrate how profit maximization requires buyers to buck less valuable logs in the assortment pricing than in free bucking, leading to deadweight losses.

4) To suggest sawn wood potential pricing, based on the analyses of the stem components. The pricing is concrete, easy to compute, transparent, incorporates the positive effect of tree size and small tapering, reduces trading cost, is less risky both to the forest owners and buyers and allows forest owners to compare biddings objectively, allows the accumulation of knowledge and standard market economy relations between price, supply and demand. It is shown how transition to this pricing can be done smoothly.

5) To estimate models for predicting the volume of the treetops which the harvester has not measured, but which is within the agreed pulp wood.

6) To make critical comments on distribution bucking and on the forestry tradition to hide information from forest owners and from researchers. I also suggest new research on optimization of bucking and saw blade settings.

The paper is not a standard research article: it is long with many components. Before the epoch of twitter publishing, the study had been a monograph. I want to keep it as single article as making several papers would cause overlapping components and it would cause extra trouble for readers to put parts spread to several papers together. The paper contains technical material which is not essential for understanding the main ideas. Readers wanting to continue the work or to apply the sawn wood potential pricing need those details. To allow selective reading, many chapters and sections start with a summary.

The key points of this paper are not scientific results. That forest owners cannot compare objectively biddings in the assortment pricing and that stem pricing does not give information for the optimization of future management, are just well-known build-in properties of the pricing methods. My pricing suggestion is not a scientific result. If an engineer designs a new ax, the ax does not need the approval of peer engineers, only acceptance of a manufacturer and clients. Assortment pricing and stem pricing have not been accepted by ‘peer reviewers’, either.

Different partners of timber trade can get different benefits from potential pricing methods. There is no scientific, value-free method to compare the benefits. Science does not tell that benefits of buyers getting deals with small effective stumpage price using misleading biddings or of forest owner associations in the bucking foreseeing business are less important than benefits produced by market economy.

The analysis of the relations of stem components and bucking optimization is standard research. It may be of interest for readers not interested in the peculiar features of Finnish timber trade. All the computations can be repeated using the open-source software Jlp22 and a script file *stem.inc* in folder *stem* at https://github.com/juhalappi/jlp22. I call readers to make a critical evaluation of my results. Critical scientists never rely on other peer reviewers than themselves.

1.3. Symbol Tables

Harvesters measure with-bark diameters. Thus, the bucking and pricing need to be based on with-bark diameters. Sawmills saw under-bark sawlogs. So, for sawmills under-bark diameters are essential. This paper is focusing on bucking and pricing. Thus, the analysis is based on with-bark diameters. Under-bark diameters need to be considered when converting with-bark volumes to the volumes and values of sawn wood.

	Bucking parameters
$L_{\min}$	Minimum length of sawlog, default 40 dm.
$L_{\max}$	Maximum sawlog length, default 55 dm.
$D_{\min}$	Minimum top diameter, default 15 cm.
$d_{\min}$	Minimum diameter of pulp wood, default 5 cm.

	Variables describing logs (upper case for with-bark variables)
d_{top}	Top diameter of sawlog under-bark.
D_{top}	Top diameter of sawlog with bark.
$f\left(d_{top}\right)$	Proportion of under-bark cross-sectional area of sawlog top or log cylinder going to sawn wood.
$u\left(u_{top}\right)$	Relative value of sawn wood coming from under-bark log with given d_{top}.

	Variables for stems
dbh	Diameter at breast height, 13 dm, measured from the stump.
d_m	Last diameter the harvester has measured.
$D(L)$	With-bark diameter at length *L*. When $L \le 13\,\text{dm}$, harvesters get $D(L)$ from official base function, here it comes from the data. Harvesters store $D(L)$ at one dm steps into stem vectors.
$L(D)$	Length of the stem to point where diameter is *D*.

	Volume components at log, stem or stand level (upper case subindex for bark)
V_{com}	With-bark volume of commercial stem up to $d_{\min}$.
V_{log}	With-bark volume of sawlog(s).
V_{pulp}	Volume of pulp wood, $V_{com} - V_{log}$.

Continued

V_{CYL}	Volume of with-bark log cylinder whose top is the cross sectional area containing bark, i.e., $0.25\pi D_{top}^2 L$ for one log with length L.
V_{cyl}	Volume of under-bark log cylinder(s), $0.25\pi d_{top}^2 L$ for one log with length L.
V_{saw}	Volume of sawn wood, including saw dust. For a log it is initially a function of log length L and under-bark top diameter d_{top}. Using a correction coefficient, it can be computed from D_{top}.
V_{sawu}	Value-weighted amount of the sawn wood where weights are proportional to the price of the sawn wood and scaled so that the total amount of V_{saw} and V_{sawu} are equal when V_{sawu} is maximized in the used data.
V_{INC}	Volume of in-chips, i.e., chips within the with-bark log cylinder, $V_{CYL} - V_{saw}$.
V_{OUTC}	Out-chips between with-bark log-cylinder and the surface of the stem, including thus bark, $V_{log} - V_{CYL}$.
V_{chip}	Volume of chips from sawlogs, $V_{INC} + V_{OUTC}$. It can be computed equivalently from under-bark variables.
V_{totc}	Total volume of chips both from sawlogs and pulp wood, $V_{chip} + V_{pulp} = V_{com} - V_{saw}$.
$V(D_1, D_2)$	With bark volume between diameters D_1 and D_2.
$V(L_1, L_2)$	With bark volume between lengths L_1 and L_2. In principle, but not in practice, this and the previous definition can lead to confusion.
V_{saw}^{max}	The maximum volume of sawn wood among all bucking schedules satisfying the bucking parameters, the sawn wood potential.

	Prices the forest owner gets (note lower case)
p_{log}	Unit price of sawlog volume in the assortment pricing.
p_{pulp}	Unit price of the pulp wood volume in the assortment pricing.
p_{stump}	Total stumpage price in the assortment pricing.
p_{saw}	Unit price of sawn wood potential V_{saw}^{max} in the sawn wood potential pricing.
p_{chip}	Unit price of total amount of chips $V_{com} - V_{saw}^{max}$ in the sawn wood potential pricing.
p_{pot}	Total stumpage price in the sawn wood potential pricing.

	Prices or values the buyers get (note upper case)
P_{saw}	The net unit price of V_{sawu} when variable production costs are subtracted from the sales price.
P_{pulp}	The unit price of pulp wood when an independent sawmill sells pulp wood to a pulp mill or energy plant.

Continued

p_{chip}	The price of chips an independent sawmill gets from chips going to pulp mills or energy plants or the 'price' a sawmill with pulp gets when 'selling' chips to the company owning both the sawmill and pulp mills, a control parameter in bucking.

Log lengths of sawlogs are assumed to step in 3 dm steps. If not stated otherwise, it is assumed that

- $L_{\min} = 40\text{ dm}$, the minimum log length;
- $L_{\max} = 55\text{ dm}$, the maximum log length;
- $D_{\min} = 15\text{ cm}$, the minimum top diameter of a sawlog.

Log lengths are given in decimeters. This is useful as harvesters store diameters at 1 dm steps. Meters would cause rounding errors in computers. Diameters are in centimeters. It is not assumed that the sawmills restrict bucking now or in the future with the above parameters. A formal analysis of the assortment pricing requires that the allowable dimensions are described precisely. The sawn wood potential pricing is based on a reference bucking. The above bucking parameters are used both in the analyses of the assortment pricing and in the sawn wood potential pricing. Ratios like V_{saw}/V_{com} are called percentages even if they must be multiplied with 100 to make them percentages.

1.4. Pricing Methods Analyzed

- Pricing methods are introduced here, analysis comes later.

In the assortment pricing, a forest owner gets price

$$p_{stump} = p_{log}V_{log} + p_{pulp}V_{pulp} \tag{1}$$

where p_{stump} is the stumpage price, V_{log} and V_{pulp} are the sawlog and pulp wood volumes the buyer bucks using the agreed dimensions and a secret price matrix, and p_{log} and p_{pulp} are their unit prices.

I suggest that a forest owner gets the sawn wood potential price

$$p_{pot} = p_{saw}V_{saw}^{\max} + p_{chip}V_{totc} = p_{saw}V_{saw}^{\max} + p_{chip}\left(V_{com} - V_{saw}^{\max}\right) \tag{2}$$

where p_{pot} is the total stumpage price in the sawn wood potential pricing, $V_{saw}^{\max}$ is the volume of sawn wood maximizing the amount of sawn wood among all bucking schedules satisfying the agreed bucking parameters, p_{saw} is its unit price, V_{totc} is the total amount of chips and bark coming both from sawlogs and the theoretical pulp wood, i.e., $V_{totc} = V_{com} - V_{saw}$ where V_{com} is the commercial volume, and p_{chip} is unit price of V_{totc}. Theoretical bucking is done using healthy parts of stems.

In stem pricing, a fixed species-dependent unit price is given for all commercial wood the harvester makes. In the size-dependent stem pricing offered by M-Group, the unit price depends on the average commercial volume of trees.

2. Data

Stem curve data of Laasasenaho (1982) are used in the analysis. Diameters and bark thicknesses were measured at relative heights of 1, 2.5, 5, 7.5, 10, 15, 20, 30, 40, 50, 60, 70, 80 and 90 percent. Diameters are interpolated linearly with one dm steps from the measured or at least 1 dm stump. The obtained stem vectors are similar as made by harvesters, except they contain also tops and stems are more regular due to the linear interpolation. The data contains trees for Scots pine, Norway spruce and birch. Birch data did not contain bark measurements. Analysis results are presented only for pine. Models for top volumes are estimated also for spruce and birch. Coefficients needed to transfer with-bark diameters into sawn wood volume and its value are computed also for spruce but not for birch. Institute of Natural Resources Finland (Luke) has allowed to put the data into https://github.com/juhalappi/jlp22.

As the data do not come from harvests, the results computed using Laasasenaho's data describe only qualitatively the relations between variables. The pricing suggestion uses stem data only for the estimation of the model for pulpwood between last cut and the minimum pulp log diameter $d_{\min}$. Saw blade setting data provided by Antti Heikkilä are used to define functions for the volume and value of sawn wood.

3. Jlp22 Software

Jlp22 is the successor of J software which was the successor of JLP software of Lappi (1992) which solves efficiently linear programming (LP) problems where a simulator simulates several treatment schedules for each stand and LP problems are formulated in terms of sums over all stands. J contained many tools for mathematical and statistical analysis. J version 2.0 published together with Reetta Lempinen (Lappi & Lempinen, 2014) made it possible optimize (somehow) simultaneously forestry and transportations to factories and factory production. Luke decided in 2017, after my retirement, to publish J as open-source software but never published it. It gave me in 2021 permission to publish and develop the software. Thereafter I have completely rewritten and enhanced it. A simpler and more efficient algorithm for the factory optimization is under implementation. The name was changed into Jlp22, because there is J programming language and Jlp22 is completely different than J. Versions are now indicated by the date when they are put into https://github.com/juhalappi/jlp22.

The most important addition for this study is function *stemopt* which can be used to compute the optimal bucking for any objective stem after stem, or to write all possible bucking schedules to the disk to be used in linear programming (LP) for the analysis of bucking possibilities for stands to be harvested. The resulting linear programming problems have the same mathematical structure as standard management planning problems for which the LP algorithm of Jlp22 was designed. Linear programming is referred only in the discussion. The optimization is done in *stemopt* function by generating for each tree all possible bucking schedules satisfying the bucking parameters and selecting the best schedule. A reader can redo all figures and the table using the script file *stem.inc* and giving command shortcut 'all'. Individual figures can be produced with shortcuts 'fig1' etc., and the table with 'table1'. Jlp22 uses in the computations also its mathematical, matrix and graphics functions. Graphics utilizes gnuplot.

4. Stem Components

A harvester divides a stem into sawlogs, pulp logs and top. Commercial volume refers here to sawlogs and pulp logs. In practice also tops may have value, at least with subsidies. It is assumed that tops may be sold with a separate deal. A sawmill divides sawlogs into sawn wood products, saw dust, chips, and bark. Saw dust is just a sawn wood product. Bark is here not separated from chips coming from sawlogs or pulp logs. The price (value) of chips is thus weighted average of bark and chip prices. It had been equally logical to put the saw dust into the chip component. A saw blade setting determines how much a log produces sawn products. The sawmill literature describes the blade settings with the ratio of log volume to the volume of sawn products. I suggest that the volume of log cylinder should be used as the reference, not the log volume. This way the joint effect of the blade setting and top diameter can be separated from the effect of the stem tapering and log length.

4.1. Sawn Wood and Its Value

- Figure 1 and Figure 2 summarize the section.

The under-bark top diameter of a sawlog d_{top} determines what blade settings are feasible. The ratio between the volume of the sawn wood and the volume of the under-bark log cylinder, V_{saw}/V_{cyl} is used here to describe blade settings. Ignoring nuances of the side boards, this ratio is equal to share of the cross-sectional area of top going to sawn wood, i.e., it is used here for all log lengths. An increasing function

$$f\left(d_{top}\right)=\frac{V_{saw}\left(d_{top}\right)}{V_{cyl}\left(d_{top}\right)} \tag{3}$$

is used to describe V_{saw}/V_{cyl}. For pricing and theoretical analysis, $f\left(d_{top}\right)$ needs to be continuous. In practical sawing, V_{saw}/V_{cyl} varies in jumps, and for a given d_{top}, there can be several feasible blade settings with different ratios. If a sawmill gets side boards only from two sides, its $f\left(d_{top}\right)$ starts to decrease when d_{top} is larger than 25 cm. Most sawmills have upper bounds for diameters. $f\left(d_{top}\right)$ assumed in the study defines a sawn wood reference value.

Figure 1 shows an example of how a blade setting determines the thickness and width of sawn products. The side boards which look larger than the top circle get sawn wood from the outside the log cylinder in the lower part of the log. However, the total amount of sawn wood is smaller than the volume of the under-bark log cylinder. Sawmills can shorten side boards to reduce the proportion of waney-edged boards. Thus, V_{saw}/V_{cyl} depends slightly on log length, but this is ignored.

Different sawn products have different values depending on the dimensions of the products and their quality. More valuable products are obtained from larger

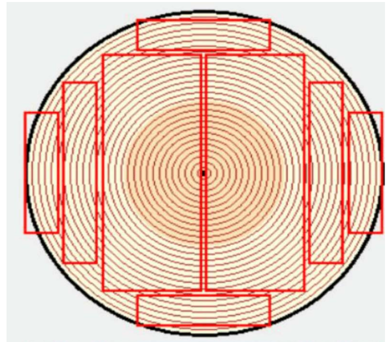

Figure 1. Sawn products produced with a blade setting. Note that the log is without bark. Figure is provided by Antti Heikkilä.

top diameters. The center yield has larger value than the side boards. The value of the sawn wood is used here in the optimization of bucking. The value depends on the top diameter of the log. The relative value of V_{saw} is described with function

$$u_{saw}\left(d_{top}\right)=\frac{V_{sawu}\left(d_{top}\right)}{V_{saw}\left(d_{top}\right)} \quad (4)$$

where V_{saw} is the volume of sawn wood and V_{sawu} is its value. Function $u\left(d_{top}\right)$ needs to be continuous in a theoretical analysis. Function $u\left(d_{top}\right)$ is scaled so that when the bucking maximizes the total value of logs using the default bucking parameters, the values add up to the total volume of the sawn wood. The scale of $u\left(d_{top}\right)$ depends on the used data. For pricing, function $f\left(d_{top}\right)$ needs to be decided, not estimated. The sawn wood potential pricing separates amount and value and uses $f\left(D_{top}\right)$ to compute the amount after making a correction needed to consider that D_{top} is used instead of d_{top}. Buyers can utilize $u\left(d_{top}\right)$ to set price for V_{saw}.

Antti Heikkilä (personal communication) gave me volume of sawn wood, its value computed from the values of obtained products using prices seen in practice, the volume of saw dust and its value for eight blade settings for five top diameters. The volumes were computed for 48.5 dm logs. One setting was from p. 52 of Räsänen et al. (2017) without values of sawn products.

Both $f\left(d_{top}\right)$ and $u\left(d_{top}\right)$ are described with the rectangular hyperbola used often to describe photosynthesis as a function of radiation. It has several parameterizations (Mehtätalo & Lappi, 2020). For a visual determination of the parameters, the expected-value parameterization of Ratkowsky (1990) going through points $\left(d_1, y_1\right)$ and $\left(d_2, y_2\right)$ is used:

$$y\left(d_{top}\right)=\frac{d_{top}\left(d_2-d_1\right)y_1y_2}{\left(d_{top}-d_1\right)d_2y_1+\left(d_2-d_{top}\right)d_1y_2} \quad (5)$$

Figure 2 shows both the data points and the obtained curves for $f\left(d_{top}\right)$ and $u\left(d_{top}\right)$. For both functions $d_1=15\text{ cm}$ and $d_2=30\text{ cm}$. Function f was fixed by $f\left(15\right)=0.72$ and $f\left(30\right)=0.81$.

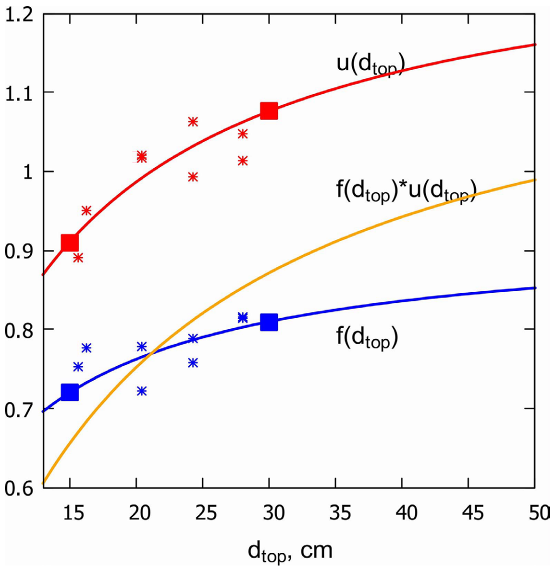

Figure 2. Blue points show V_{saw}/V_{cyl}, (V_{saw} is the volume of sawn wood, and V_{cyl} volume of without-barklog cylinder) for six blade settings and 48.5 dm logs. Red points are for ratio V_{sawu}/V_{saw} where V_{sawu} is the relative value of obtained sawn wood volume. d_{top} is the without-bark top diameter of the sawlog. Squares show the points (d_1, y_1) and (d_2, y_2) in Equation (5) used to define $f(d_{top})$ and $u(d_{top})$ (Equations 3 and 4) and used via $f(D_{top})$ and $u(D_{top})$ in Equations (6) and (7). Orange curve shows the value of log combining the volume of sawn wood, V_{saw}, and $u(d_{top})$.

In bucking and sawing V_{saw} needs to be computed from with-bark top diameter D_{top}. It was found that it can be very well predicted using D_{top} as the argument in $f(\,)$ and using a correction coefficient which can be computed by regressing $f(d_{top})d_{top}^2$ on $f(D_{top})D_{top}^2$ without intercept.

The sawn wood volume V_{saw} (dm³) of a sawlog with length L (dm) and with-bark top diameter D_{top} (cm) can then be computed using

$$V_{saw}(L, D_{top}) = c_{sp} 0.25\pi f(D_{top}) D_{top}^2 L \tag{6}$$

where c_{sp} is s species dependent coefficient. Regression provided $c_{pine} = 0.900$ and $c_{spruce} = 0.873$ ($R^2 = 0.994$ for both regressions). Measurements such that $L(D) \geq 37$ dm and $D(L) \geq 14$ cm were used. The coefficients agree well with the rule of thumb that bark is 10% of the log volume.

The value weighted sawn wood volume V_{sawu} was scaled (y_1 and y_2 multiplied with the same number) so that when V_{sawu} is maximized with default bucking parameters, the average V_{sawu} is equal to average V_{saw}. The scaling takes the with-bark correction automatically into account. Thus

$$V_{savu}(L, D_{top}) = u(D_{top}) V_{saw}(L, D_{top}) \tag{7}$$

The y-values for u were after scaling: $u(15) = 1.012$ and $u(30) = 1.1956$.

Multiplying these with c_{pine} provides the read squares in Figure 2, i.e. $y_1 = 0.9105$ and $y_2 = 1.076$.

4.2. Log Cylinder, Sawn Wood, and Chips

- Figures 3-6 summarize this section. Flat tops of curves in Figure 4 and Figure 5 explain the controversy of forest owners and buyers in the assortment pricing.

Let us then look at the components of stems. The basic set-up can be seen from Figure 3 for a stem producing one sawlog. The log is assumed to be cut at the length (not 'height') where $D(L) = 16\ \text{cm}$, leading to L = 50 dm. The blue rectangle shows the with-bark log cylinder. The log cylinder is decomposed into sawn wood V_{saw} (including saw dust) and with-bark in-chips V_{INC} using equation (6). Within a sawlog, volume outside the with-bark log cylinder provides out-chips, V_{OUTC} which contains bark.

Figure 4 (left) shows how V_{CYL}, V_{saw} and V_{sawu} depend on the cutting length L for the stem of Figure 3. Length L producing maximal V_{sawu} would be optimal if chips have no value. The effect of the value of chips is demonstrated in Figure 4 (right) showing the ratio of $P_{saw}V_{saw} + P_{chip}V_{totc}$ to maximal $P_{saw}V_{saw}$ for different values of P_{chip}/P_{saw}. Using V_{saw} instead of V_{sawu} makes the interpretation simpler.

Figure 5 shows an example of a stem where maximization of V_{saw} produces two logs and Figure 6 shows examples how V_{saw} depends on V_{log} of the last log if all other logs are 40 dm long. The curves are drawn to up to L where $D(L) = 15\ \text{cm}$.

Volumes are integrated cross-sectional areas. Thus $D(L)$ does not show properly the effect of stem curve on the volume. The green curve in Figure 3 shows the (scaled) cross-sectional area, the green curves in Figure 4 (left) and Figure 5 show the integrated cross-sectional area, i.e., volume.

Remarks from Figures 3-6:

- Curves for V_{CYL}, V_{saw} and V_{sawu} have similar shapes and flat tops. V_{saw} has maximum at slightly smaller L than V_{cyl} owing to $f(D_{top})$, and V_{sawu} has maximum at smaller L than V_{saw} owing to $u(D_{top})$.
- When moving left from the point maximizing V_{sawu}, V_{sawu} decreases slowly but V_{log}, *and* stumpage price in the assortment pricing, decreases rapidly. Thus, the profit maximization requires the buyer to buck smaller V_{sawu} in the assortment pricing than possible.
- The ratio of the green curve to the red curve in Figure 4 (left) and Figure 5 is proportional (approximately due to saw dust) to V_{log}/V_{saw} which sawmills use to describe the yield in sawing. It combines the effects of log length, stem curve and blade settings. An analysis of these effects using V_{log}/V_{saw} is difficult.
- Ratio V_{CYL}/V_{log} describes well the effect of tapering on the sawn wood yield, also for stems producing several sawlogs.

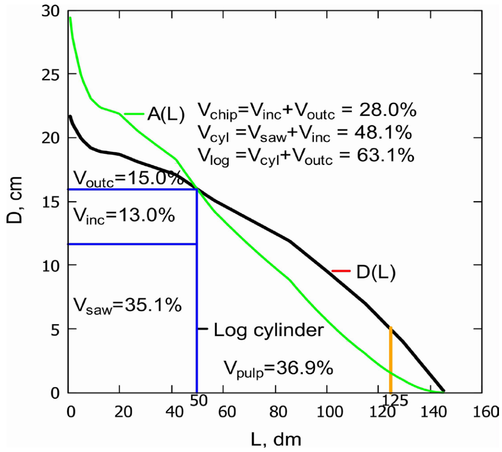

Figure 3. A typical one-log stem of Scots Pine. The x-axis L is the length from the stump. The black curve is $D(L)$ (scale in y-axis). The blue rectangle is the log cylinder of the log cut at L = 50 dm where $D(L) = 16$ cm. The green curve is the cross-sectional area $A(L)$ scaled to coincide with the top right corner of the log cylinder. The vertical orange line at $L = 125$ dm shows the length of the commercial stem, i.e., $D(125) = 5\text{ cm} = d_{\min}$, $d_{\min}$ being the diameter limit for pulp wood. In the legend, $V_{\log}$, V_{cyl}, V_{saw} and V_{pulp} are volume percentages of the sawlog, log cylinder, sawn wood and pulp wood, respectively, from the commercial volume V_{com}.

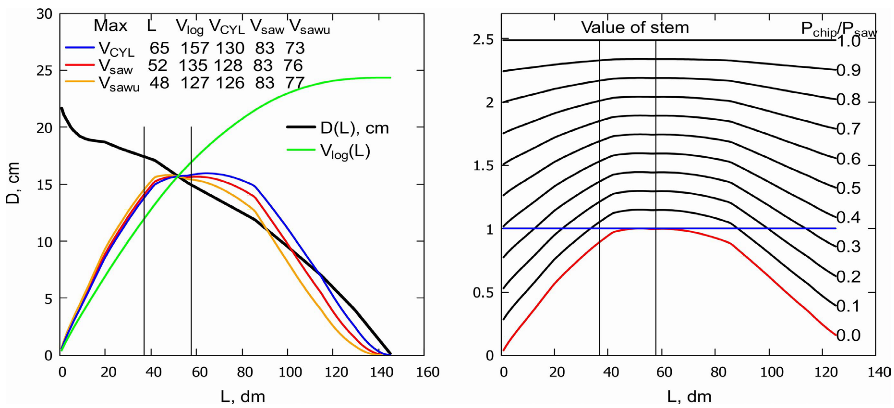

Figure 4. Left subfigure shows volume components $V_{\log}$, V_{cyl}, and V_{saw} for the stem shown in **Figure 3** if the sawlog is cut at length L on the x-axes. The curves are scaled so that they cross $D(L)$ at L = 50 dm, which was the log length in **Figure 3**. V_{sawu} refers to value weighted sawn wood volume which is scaled similarly. The legend shows what L, V_{log}, V_{cyl}, V_{saw} and V_{sawu} would result if V_{cyl}, V_{saw} or V_{sawu} is maximized. Right subfigure shows the ratio of the value of the stem computed with $P_{saw}V_{saw} + P_{chip}V_{totc}$ to the optimal value of $P_{saw}V_{saw}$ for different values of P_{chip}/P_{saw} shown at the end of each curve. Because the optimal sawn wood percentage V_{saw}/V_{com} is smaller than 50%, the curve for $P_{chip} = P_{saw}$ is higher than 2. Red curve on the right is the same V_{saw} curve as on the left. The range of feasible values of L is shown for both figures for $L_{\min} = 37$ dm, $L_{\max} = 58$ dm, and $D_{\min} = 14$ cm.

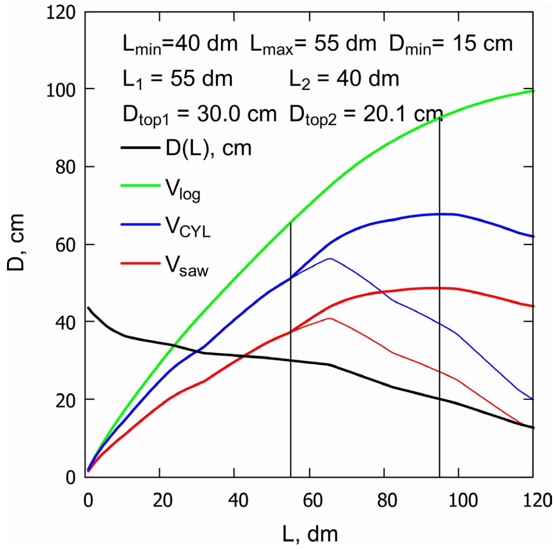

Figure 5. An example of a stem where maximization of V_{saw} produces two logs. The thin blue and red lines show how V_{cyl} and V_{saw} had continued if the first log had not been cut at 55 dm which was the maximum length of sawlogs, $L_{\max}$. Note that D_{top} of the second log was larger than $D_{\min}$ = 15 cm, the assumed minimum top diameter, and log length 40 dm had also allowed a longer log. Thus, maximization of V_{log} and V_{saw} are in conflict. Note the sharp bend of V_{cyl} and V_{saw} on the thin lines. The bend of $D(L)$ is mild at the same L. The volume curves are at the same scale, which is different from $D(L)$.

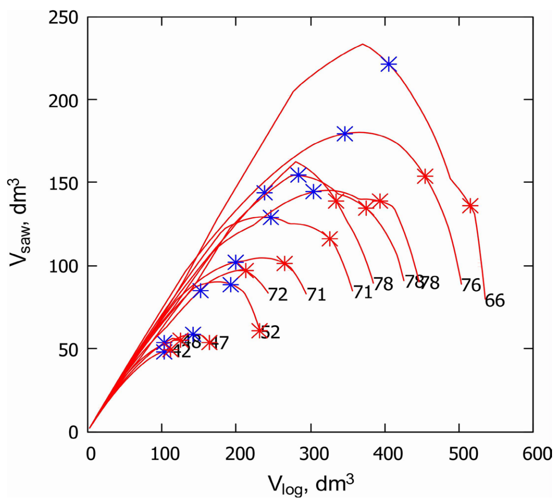

Figure 6. Randomly selected examples of V_{saw} as a function of V_{log} for the last log when making initially logs using $L_{\min}$ = 40 dm. End points of lines are obtained using L (shown with number) where $D(L)$ = 15 cm. Blue points are for L = 37 dm, the smallest $L_{\min}$ used in the paper, and red points are for L which is the minimum of 58 dm and L producing $D(L)$ = 15 cm. The number at the top of a curve shows the log length where the maximum sawn wood is obtained.

- In free bucking, sawmills might consider making the last log slightly (e.g. 3 dm) longer than seems optimal at the harvest time. The final length could then be determined at the sawmill. Pertti Holmila (personal communication) told that this idea may not be feasible from the viewpoint of sawing technology.
- When the value of chips is considered, value curves are even more flat (Figure 4 (right)). Due to dimension restrictions, only a narrow window of such curves is relevant for a given stem.
- Assortment pricing causes conflicts because V_{log} is the main concern of the forest owner but V_{sawu} is almost independent of V_{log} at potential lengths. In Figure 4 (left), V_{CYL}, V_{saw} and V_{sawu} have almost equal values when any of them is maximized, but values of L and V_{log} are much different. After reaching the top area of V_{sawu}, the sawlog volume is always expensive for the buyer, but it may not be valuable.
- Figure 6 shows that the requirement that the buyer should cut the last log up to the agreed diameter limit would decrease the sawn wood volume significantly and the value-weighted sawn wood even more. Restrictions for log lengths reduce the potential decrease.

4.3. Relations of Volume Components

- Figure 7 shows how V_{saw}/V_{log} increases with log size (D_{top} and V_{log}). The sawn wood potential pricing takes this into account, the assortment pricing ignores. Figure 8 is for stem components.

This section describes some empirical relations between stem components in the pine data when logs are bucked maximizing V_{sawu}, the value-weighted amount of sawn wood using the default bucking parameters $L_{\min} = 40\text{ dm}$, $L_{\max} = 55\text{ dm}$ and $D_{\min} = 15\text{ cm}$. Figure 7 shows how relative components depend on D_{top}, the top diameter of log, and log volume V_{log}. Figure 8 shows share of components from V_{com} with respect to *dbh* (left) and V_{com} of the stem (right).

Remarks:

- Curves of V_{sawu}/V_{log} and V_{sawu}/V_{log} in Figure 7 are concave. Curves evaluated at average values of D_{top} and log-wise V_{log} give too high predictions for average percentages, as well as curves of V_{sawu}/V_{com} in Figure 8 at average values of *dbh* and stem-wise V_{log}. Average log-wise or stem-wise percentages are, however, usually of no interest. Average of stem-wise V_{saw}/V_{com} does not tell what is V_{saw}/V_{com} for V_{saw} and V_{com} of the stand.
- M-Group probably considers the thin orange line in Figure 8 (right) in its size-dependent stem pricing. The company gets a more precise analysis from absolute value figures like subsequent Figure 17.
- In the sawn wood potential pricing, the unit price of the commercial wood depends on the ratio $V_{saw}^{\max}/V_{com}$ where $V_{saw}^{\max}$ and V_{com} are the sums of potential sawn wood and the commercial volume in the whole stand. Subsequent Figure 17 shows how values of $V_{saw}^{\max}$ and V_{com} depend on *dbh*. These functions can be used to estimate $V_{saw}^{\max}$ and V_{com} from the *dbh* distribution. Figure 8 (right) cannot be utilized.

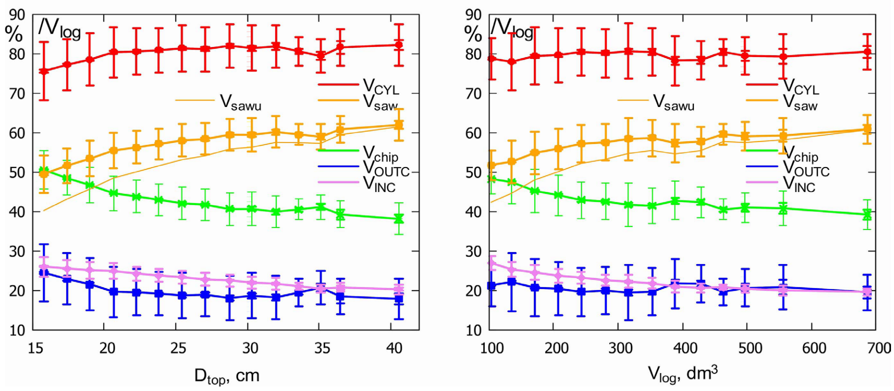

Figure 7. Relative volumes of log components from the total log volume as a function of the log size, on the left with respect to D_{top}, the value-weighted sawn wood V_{sawu} is also drawn with thin orange line. As log length has trivial effect on the components, all logs are 40 dm long (the lower part of longer logs is ignored). The thin error bars show the standard deviation in each class, the thick error bar shows the standards error of mean computed as $sd/\sqrt{n}$. When D_{top} and L are fixed then V_{cyl}, (12) V_{saw} and V_{inc} are constants. The variation of the share of V_{cyl} and V_{saw} is caused by the variation of V_{log}. In grouping, 20 classes of equal length are first formed, and then classes are merged until each class has at least seven observations.

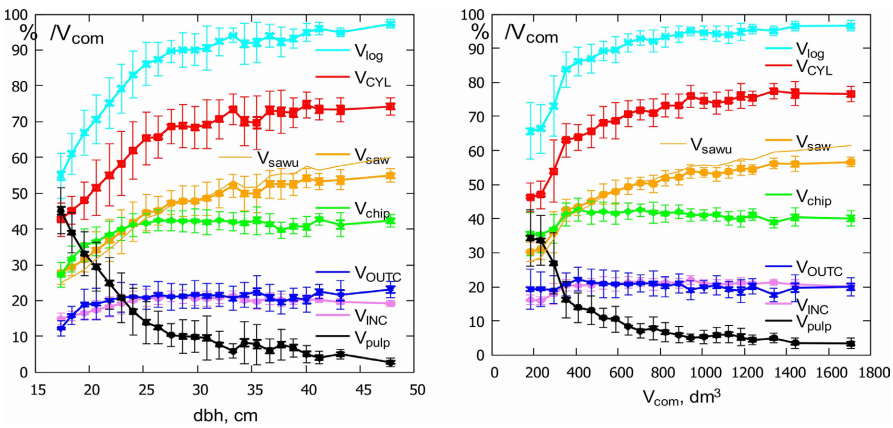

Figure 8. Relative volumes of log components from the total commercial volume V_{com} of the stem as a function of *dbh* (left) and commercial volume V_{com} (right). The relations are more nonlinear than log-wise percentages in **Figure 7**. Note that $100 - V_{saw}$ is the share of total amount of chips, V_{totc}. Also V_{sawu} is drawn with thin orange line. Error bars as in **Figure 7**.

- **Figure 7** and **Figure 8** were computed maximizing V_{sawu}, i.e., ignoring value of chips and assuming free bucking. The effects of the value of chips and the pricing method are analyzed in Section 5.5.

In the assortment pricing the price is given for the sawlog volume without considering the size of the tree. In the sawn wood potential pricing, price is given to V_{saw} without considering that larger top diameters provide more valuable sawn products. The close relation between thin and thick orange lines in **Figure**

8 shows that V_{saw} considers most part of the utility of larger trees. Further details are given later.

5. Bucking

5.1. Generating All Bucking Schedules

- A simple algorithm can generate all possible bucking schedules. The best schedule can be selected using any objective function.

The bucking can be optimized with respect to any log variables by selecting the best bucking schedule among all feasible bucking schedules. A bucking schedule tells the lengths of logs. The algorithm below is used to generate all bucking schedules obeying bucking parameters $L_{\min}$, $L_{\max}$ and $D_{\min}$, and assuming that sawlogs step in 3 dm steps. If it is not allowed to leave a possible sawlog to the pulp wood, the following algorithm implemented in *stemopt* in Fortran 90 generates all possible schedules:

```
do ilog1=Lmin,min(loglentot,Lmax),3
  loglen(1)=ilog1    ! loglen = length of the log
  loglenc(1)=ilog1 ! loglenc = cumulative length
  nlog=1                 ! nlog = number of logs
  if(loglentot-loglenc(1).ge.Lmin)then
    do ilog2=Lmin,min(loglentot-loglenc(1),Lmax),3
      loglen(2)=ilog2
      loglenc(2)=loglenc(1)+ilog2
      nlog=2
      if(loglentot-loglenc(2).ge.Lmin)then

Nesting goes up to level 6

      else
        call compute()    ! two logs are possible nlog=2
      endif !if(loglentot-loglenc(2).ge.Lmin)
    enddo !ilog2=Lmin,min(loglentot-loglenc(1),Lmax),3
  else
    call compute() ! only one log is possible nlog=1
  endif !if(loglentot-loglenc(1).ge.Lmin)    enddo
```

The subroutine *compute* computes the objective and updates the current solution if the schedule is better than current optimum. In the data sets of Luke and with the used parameters the maximum number of logs was 5. The loops go now to level 6. More levels can be easily added. When generating all possible schedules allowing the pulp part to contain potential sawlogs, then *if-then-else* structures are dropped.

Researchers have developed bucking algorithms based on recursion or dynamic programming (e.g. Gronding, 1998). The above algorithm, used e.g. in Näsberg (1985), is simple and fast. Bucking optimization of 2169 trees in Luke's pine data took 0.14 secs of CPU time in my ancient laptop, when all possible output variables were produced. It is possible, also in *stemopt*, to put further restrictions to allowable logs. Setting small values to $L_{\min}$ and $D_{\min}$, and a large value to $L_{\max}$ and putting restrictions to allowable logs, allows arbitrary log dimension definitions.

All variable needed in this paper are produced by *stemopt*. The objective can be given in terms of these variables either as a one mathematical statement possibly also containing matrix algebra, or using a separate function which can loop over logs. The optimization means selecting the schedule with the largest value of the objective. If the *compute*-subroutine is asked to write the schedule variables into the disk, the resulting data has the same structure for which the linear programming algorithm of Jlp22 is designed. Trees correspond to stands in management planning problems.

5.2. Influence of Bucking Parameters and Simple Objectives

- **Figure 9** shows that buyers have much freedom when deciding the sawlog percentage in assortment pricing.

Bucking parameters determine the range of possible bucking results for any objective. The assortment pricing gives much freedom to the buyer to decide the sawlog percentage. V_{log}/V_{com} and V_{saw}/V_{com} were computed for eight bucking parameter combinations and seven objectives. The objectives were: V_{saw}, V_{sawu}, V_{saw}/V_{log}, $-V_{log}-\varepsilon V_{saw}$, $-V_{log}+\varepsilon V_{saw}$, $V_{log}+\varepsilon V_{saw}$, $V_{log}-\varepsilon V_{saw}$. Maximization of $-V_{log}$ means minimization of V_{log}. Same V_{log} can be obtained with different log lengths when more than one log is bucked. The ε-component with small ε was used to maximize or minimize V_{saw} for the same maximal V_{log}. If the capacity restricts the production instead number of available stands, an independent sawmill may consider maximizing V_{sawu}/V_{log}. Objectives depending on the stumpage price and prices of products are discussed later. In the assortment pricing, a forest owner would like that the buyer would maximize V_{log} and fears that the buyer minimizes V_{log}.

Figure 9 shows V_{log}/V_{com} and V_{saw}/V_{com} for other objective than V_{sawu}, $-V_{log}-\varepsilon V_{saw}$ and $V_{log}-\varepsilon V_{saw}$ which would lead poorly separated lines. All objectives are shown for the default bucking parameters in **Table 1**.

Table 1. V_{log}/V_{com}, V_{saw}/V_{com} and V_{sawu}/V_{com} multiplied with 100 when objectives shown in the top are maximized.

Max $/V_{com}$	V_{saw}	V_{sawu}	$\frac{V_{saw}}{V_{log}}$	$-V_{log}-\varepsilon V_{saw}$	$-V_{log}+\varepsilon V_{saw}$	$V_{log}+\varepsilon V_{saw}$	$V_{log}-\varepsilon V_{saw}$
V_{log}	90.3	89.5	82.1	78.1	78.1	92.3	92.3
V_{saw}	48.6	48.5	45.9	42.0	42.50	46.6	47.6
V_{sawu}	48.4	48.4	46.3	42.3	42.8	46.0	47.2

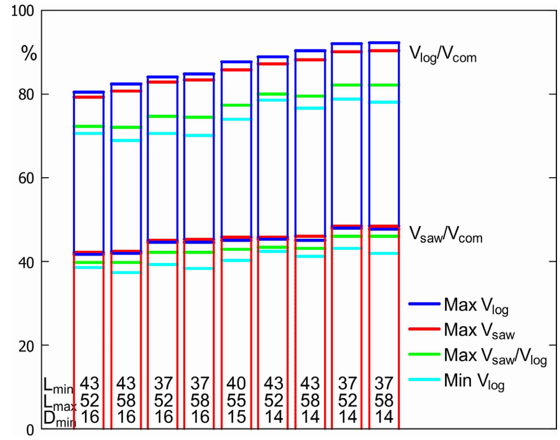

Figure 9. Bucking results for different bucking parameters shown in the bottom of the columns. The horizontal lines in the middle show the percentage of the volume of sawn wood V_{saw} from the total commercial volume V_{com} for objectives shown in the legends. The upper horizontal lines show the percentages of V_{log} from V_{com}. Columns are ordered according to V_{saw}/V_{com}. The middle column is for the default bucking parameters.

Remarks:

- The differences in sawn wood are smaller than differences in sawlogs. V_{log} can be reduced with small sacrifice in V_{saw}. Buyers can utilize this in the assortment pricing.
- The minimization of V_{log} is a rational fear for a forest owner when a sawmill with pulp uses high price for chips in the bucking algorithm as seen in Sections 5.6 and 5.8.

In the assortment pricing, different buyers use many other dimensions than present in **Figure 9**. Buyers can determine feasible log dimensions so that they are not forced to buck large V_{log}. With a given set of allowable dimensions, the secret price matrices of buyers can give a wide range of sawlog percentages.

V_{saw} was maximized in the bucking for the same bucking parameter combinations as used above. Regression of total $100V_{saw}^{\max}/V_{com}$ on the bucking parameters produced the following equation. The bucking parameters were scaled to −1, 0, and 1.

$$100\frac{V_{saw}^{\max}}{V_{com}} = 50.5 - 1.55L_{\min} + 0.09L_{\max} - 1.91D_{\min} \tag{8}$$

RMSE of Equation (8) was 0.15 and t-values for the coefficients were −29.2, 1.7 and −36.9. Even if the regression equation is not estimated from data satisfying standard assumptions, the t-values and coefficients show that V_{saw} depends on $L_{\min}$ and $D_{\min}$ in an anticipated way and $L_{\max}$ is less important. If different buyers would use the sawn wood potential pricing but different bucking parameters, Equation (8) or its tuned version with quadratic terms could be used to make biddings comparable.

5.3. Bucking Objectives in Free Bucking

- In free bucking, buyers optimize value of all logs. For independent sawmills, pulp logs are not important.

When optimizing bucking in each stand, only variable production costs need to be considered when computing the values of the produced logs. The harvest and transportation costs both to the nearest road and from there to the sawmill or pulp mill are variable costs. The pulping process does not allow decreasing fluently the production level and production costs. Sawmills are more flexible. When the production is decreased, a sawmill can save e.g. in the cost of electricity and the cost of drying sawn wood but not very flexibly in the cost of labor. At the theoretical level of this paper, also the production costs at sawmills can be treated as fixed.

Utilizing the stem components, we may assume that a sawmill gets the net income:

$$U = P_{saw}V_{sawu} + P_{chip}V_{chip} + P_{pulp}V_{pulp} - p_{stump} \tag{9}$$

where P_{shaw}, P_{chip} and P_{pulp} are the net prices of V_{sawu}, V_{chip} and V_{pulp}, respectively, after subtracting the variable costs and p_{stump} is the stumpage price. p_{stump} is always dependent on the stem properties. But if p_{stump} does not depend on the actual bucking, Equation (9) can be maximized by maximizing

$$U_f = P_{saw}V_{sawu} + P_{chip}V_{chip} + P_{pulp}V_{pulp} \tag{10}$$

The subscript f of U refers to free bucking. We may assume that both sawmill types are in the same sawn wood market and have the same variable costs for sawmill production, even if sawmills with pulp may have some logistic savings if a sawmill is close to a pulp mill. Generally, P_{saw} may be the same for both. As results are computed for different values of P_{saw}, it is possible to compare one P_{saw} for independent sawmills to another P_{saw} for sawmills with pulp.

For an independent sawmill P_{chip} is the price of chips it gets from the market. For a sawmill with pulp, P_{chip} in Equation (10) is the value the company gives from chips coming from its own harvests. If there were perfect competition for chips, P_{chip} for a sawmill with pulp would basically have the same value as an independent sawmill gets from the market. Because the competition may not be perfect (Kallio, 2001), P_{chip} may be higher for sawmills with pulp. In the following computations P_{chip} is allowed to approach the value of sawn timber.

P_{pulp} has evidently a different value for sawmills with pulp than for independent sawmills. As sawmills with pulp process V_{pulp} they are expecting to get also profit for V_{pulp}. Chipping pulp wood and chipping the chip component V_{chip} of sawlogs is so similar and the longer fibers coming from sawlogs are not much more valuable. We may assume that $P_{pulp} = P_{chip}$ for sawmills with pulp. As independent sawmills just transmit pulp wood to pulp mills, we may assume that they do not get any operating loss or operating profit from pulp wood, i.e., that for them $P_{pulp} = 0$. Thus Equation (10) is interpreted to mean for independent sawmills

$$U_{fi} = P_{saw} V_{sawu} + P_{chip} V_{chip} \tag{11}$$

and for sawmills with pulp

$$U_{fp} = P_{saw} V_{sawu} + P_{chip} V_{totc} \tag{12}$$

where V_{totc} is the total amount of chips coming both from sawlogs and pulp wood, i.e. $V_{totc} = V_{chip} + V_{pulp}$. Currently, independent sawmills may get $P_{chip} \approx 40\ €/\text{m}^3$.

5.4. Bucking Objectives in Assortment Pricing

- In assortment pricing, buyers maximize the difference between the value of logs and the stumpage price.

In assortment pricing the stumpage price is dependent on the bucking. Profit maximization requires independent sawmills to maximize:

$$U_{ai} = P_{saw} V_{sawu} + P_{chip} V_{chip} - p_{log} V_{log} \tag{13}$$

and sawmills with pulp may maximize

$$U_{ap} = P_{saw} V_{sawu} + P_{chip} V_{totc} - p_{log} V_{log} - p_{pulp} V_{pulp} \tag{14}$$

Computations are done assuming that $p_{log} = 75\ €/\text{m}^3$ and $p_{pulp} = 25\ €/\text{m}^3$, which agree approximate the price statistics of Northern Savo some time ago.

5.5. Comparing Free Bucking and Assortment Pricing

- When P_{chip}, value of chips increases, sawlog percentage decreases fast in the assortment pricing for sawmills with pulp but V_{saw}, sawn wood volume decreases only a little. In free bucking, log percentage decreases less. Independent sawmills should increase V_{log} when P_{chip} increases, but less in the assortment pricing.

Figure 10 shows V_{log}/V_{com} (upper curves) and V_{saw}/V_{com} (lower curves) as a function of P_{chip} for three values of P_{saw} for independent sawmills (left) and sawmills with pulp (right). Cyan lines show the minimum and maximum levels. Note that V_{saw} decreases the same amount as V_{totc} increases.

Figure 11 shows the average log length with respect to P_{chip}. The average log length is inversely related to V_{saw}/V_{log}: long logs imply small V_{saw}/V_{log}. For independent sawmills, the number of logs decreases slightly when $P_{chip} > 200$ and for sawmills with pulp very rapidly in the assortment pricing when P_{chip} approaches P_{saw}.

Remarks:

- In free bucking, log lengths, V_{log}, V_{chip} and V_{saw}/V_{log} go in the different direction for sawmills with pulp than for independent sawmills when P_{chip} increases.
- If the bucking parameters allow a wider range of log dimensions than used in **Figure 10** (left), high chip prices would reduce sawn wood also for independent sawmills more than shown in the figure.

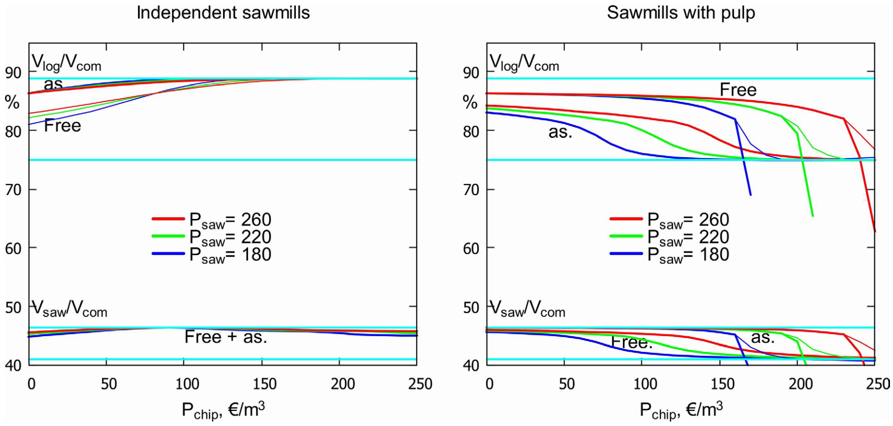

Figure 10. Dependency of V_{log}/V_{com} (upper curves) and V_{saw}/V_{com} (lower curves) on P_{chip} for three values of P_{saw} for both sawmill and pricing types. The right plot shows the behavior of V_{log}/V_{com}. Because $V_{com} = V_{saw} + V_{totc}$ and the volume of commercial volume V_{com} is fixed, decrease of sawn wood V_{saw} implies increase of the total volume of chips V_{totc} with the same amount. Thus, distance of V_{saw}/V_{com} from the lower and upper margin should be considered simultaneously.

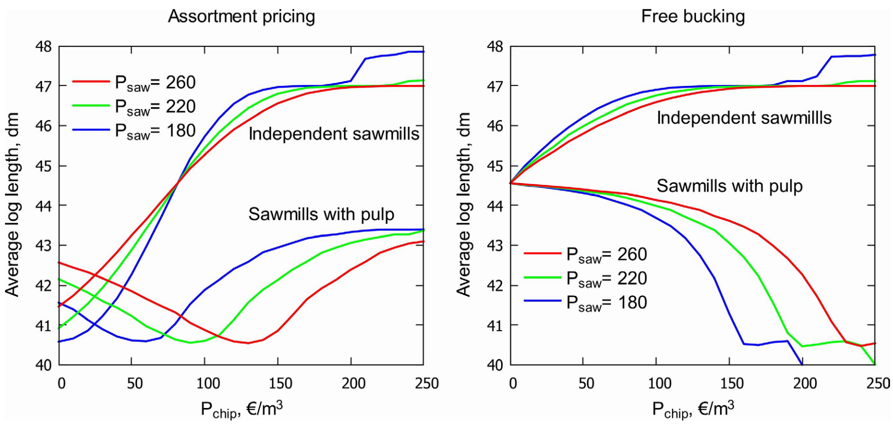

Figure 11. Average log length as a function of P_{chip} when Equations (11) - (14). Currently, independent sawmills may get $P_{chip} \approx 40$ €/m³. Note that log lengths approach the used $L_{\min} = 40$ but are much lower than $L_{\max} = 55$ dm.

- In the assortment pricing, it is initially profitable for sawmills with pulp to increase amount of chips by bucking shorter logs and move chips from the expensive sawlog to cheap pulp logs with very slight decrease of V_{sawu} and V_{saw}. Recall that V_{saw} decreases as much as V_{totc} increases, and decrease is slight due to flat tops of curves. But when $L_{\min}$ and the requirement that the pulp part cannot contain feasible sawlogs makes shortening of logs infeasible, also sawmills with pulp need increase the amount of chips coming from the sawlogs by increasing log lengths. A figure showing number of logs as a function of P_{chip} looks like the middle subfigure of subsequent **Figure 15**.
- If the common view that sawmills with pulp buck significantly smaller V_{log} than independent sawmills, is true, the competition at the chip market is not

perfect. In perfect competition it does not make difference whether chips are bought or bucked from own harvests.

5.6. Stumpage Price in Both Pricings and Dead Weight Loss in Assortment Pricing

- Figure 12 shows that when the price of chips increases, the profit maximization requires sawmills with pulp to decrease the stumpage price. In free bucking they need to buck smaller sawlog percentages only with very high chip prices. The interpretation of the dead weight loss (Figure 13) is more complicated for sawmills with pulp.

Let U-values refer to optimal values when Equations (11) - (14) are maximized. In economics inefficiency resulting when market economy does not work properly is called deadweight loss. In the assortment pricing, deadweight loss for an independent sawmill is

$$DWD_i = U_{fi} - U_{ai} \tag{15}$$

and for sawmill with pulp the deadweight loss is

$$DWD_p = U_{fp} - U_{ap} \tag{16}$$

Before finding the proper term in economics, I called dead weight loss 'black hole', which might be easier for non-economists to understand.

Figure 12 compares the stumpage prices both for the independent sawmills and sawmills with pulp to stumpage prices which would result if the buyers would buck according the optimal free bucking and would pay the same p_{log} and p_{pulp} for the obtained sawlogs and pulp wood.

Some comments made in previous section could be repeated. Following additional remarks can be made:

- The profit maximization requires sawmills to reduce V_{log} to reduce the stumpage price in the assortment pricing.
- In free bucking, forest owners could not anticipate increasing their incomes as much as the theoretical decrease of the stumpage price indicates. In free bucking forest owners are not paid according to the bucking the harvester makes. There is no reason to assume that forest owners would get initially more or less money in free bucking. In the long run, forest owners, industry and the society would get benefits provided by the market economy.
- It should be recalled that the data do not come from harvests. The size distributions in practical harvests are different than in the data used. The below analysis of whether a buyer should make two logs or one log from potential two-log stems may indicate that the reductions of V_{log} and stumpage prices may be much larger than indicated in Figures 10-12.
- The dead weight losses in Figure 13 are not very large. I argue later that other losses to the society are more important. The fact that in the assortment pricing the profit maximization requires to buck less than optimal logs may demonstrate that there is something fundamentally irrational in the assortment pricing.

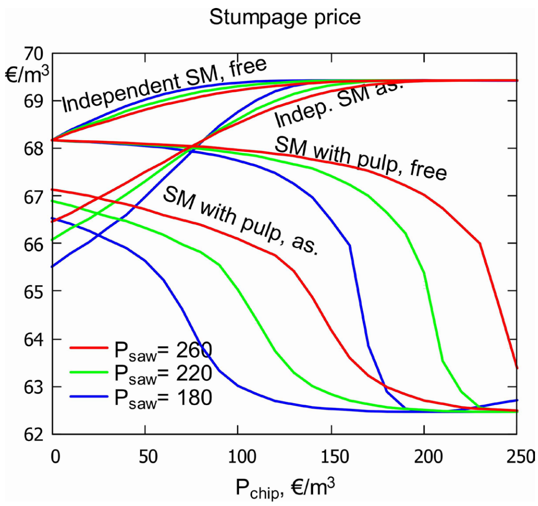

Figure 12. 'Stumpage price' $p_{log}V_{log} + p_{pulp}V_{pulp}$ in free bucking when Equation (11) is maximized for independent sawmills ('Independent SM, free') and Equation (12) is maximized for sawmills with pulp ('SM with pulp, free'). Real stumpage price $p_{log}V_{log} + p_{pulp}V_{pulp}$ in assortment pricing after maximizing Equation (13) for independent sawmills ('Indep. SM, as.') and after maximizing Equation (14) for sawmills with pulp ('SM with pulp') with respect P_{chip}. If the buyers would buck similarly as would be optimal in free bucking and would anyhow pay p_{log} for V_{log} and p_{pulp} for V_{pulp}, the forest owner would get stumpage price shown with 'free' curves.

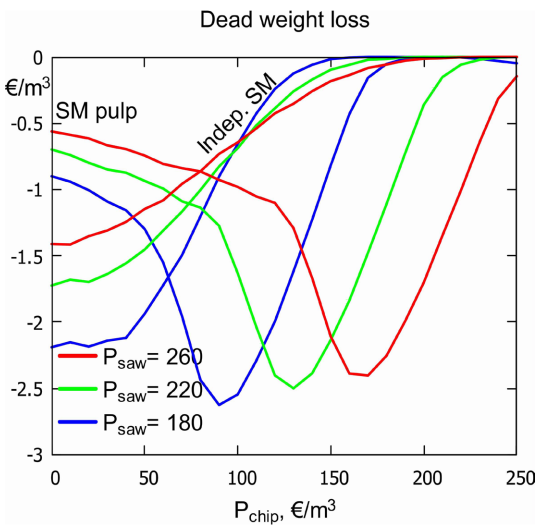

Figure 13. Dead weight loss computed using Equation (15) for independent sawmill ('Indep. SM') and Equation (16) for sawmill with pulp ('SM pulp') with respect to the price of chips P_{chip}.

I do not have competence to fully understand the economic interpretation of P_{chip} in the Equations (12) and (14) for sawmills with pulp even if the bucking optimization clearly needs such term. A reason for the difficulty is that P_{chip} can be interpreted both as a marginal utility and a control parameter for controlling the flow of chips.

5.7. Cost of Chips

- When the price of chips increases in the bucking objective, the buyer gets more chips. For independent sawmills cost more in the assortment pricing. Figure 14 (right) shows that sawmills with pulp can make forest owners to pay part of additional chips in the assortment pricing.

Both in free bucking and in the assortment pricing, increasing P_{chip} produces larger V_{chip} for independent sawmills. For sawmills with pulp, increasing P_{chip} produces larger V_{totc}. It is straightforward to analyze how much the increase of V_{chip} or of V_{totc} costs, because P_{saw}, p_{log} and p_{pulp} determining the cost are quite well defined. The volumes are expressed as a function of P_{chip}. The cost for independent sawmills in free pricing is (when Equation (11) is optimized):

$$c_{fi} = \frac{P_{saw}\left(V_{sawu}(0) - V_{sawu}\left(P_{chip}\right)\right)}{V_{chip}\left(P_{chip}\right) - V_{chip}(0)} \tag{17}$$

The cost for independent sawmills in the assortment pricing is (Equation (13)):

$$c_{ai} = \frac{P_{saw}\left(V_{sawu}(0) - V_{sawu}\left(P_{chip}\right)\right) + p_{log}\left(V_{log}\left(P_{chip}\right) - V_{log}(0)\right)}{V_{chip}\left(P_{chip}\right) - V_{chip}(0)} \tag{18}$$

The cost for sawmill with pulp in free bucking is (Equation (12)):

$$c_{pi} = \frac{P_{saw}\left(V_{sawu}(0) - V_{sawu}\left(P_{chip}\right)\right)}{V_{saw}(0) - V_{saw}\left(P_{chip}\right)} \tag{19}$$

The cost for sawmill with pulp in the assortment pricing is (Equation (14):

$$c_{ap} = \frac{P_{saw}\left(V_{sawu}(0) - V_{sawu}\left(P_{chip}\right)\right) - \left(p_{log} - p_{pulp}\right)\left(V_{saw}(0) - V_{saw}\left(P_{chip}\right)\right)}{V_{saw}(0) - V_{saw}\left(P_{chip}\right)} \tag{20}$$

Note that differences between V terms are presented in the direction producing positive values. The difference between pricing methods cannot be inferred from the p_{log} in the numerator because the denominator also is different.

Remarks based on Figure 14:

- That cost of chips is smaller than P_{chip} is natural requirement for running business with chips.
- Independent sawmills pay more for chips in the assortment pricing than in free bucking.
- When $P_{chip} > 100\ €/\mathrm{m}^3$ sawmills can make forest owner pay for the additional chips. The larger P_{saw} is, the higher P_{chip} must before this happens. For some values of P_{chip} the additional chips cost more in free bucking.
- The assumption of perfect competition of chips implies that sawmills with pulp apply approximately same P_{chip} in the bucking as they pay for independent sawmills for their chips. A company may need rapidly more chips to maintain pulping process and there may a limited amount of chips at the

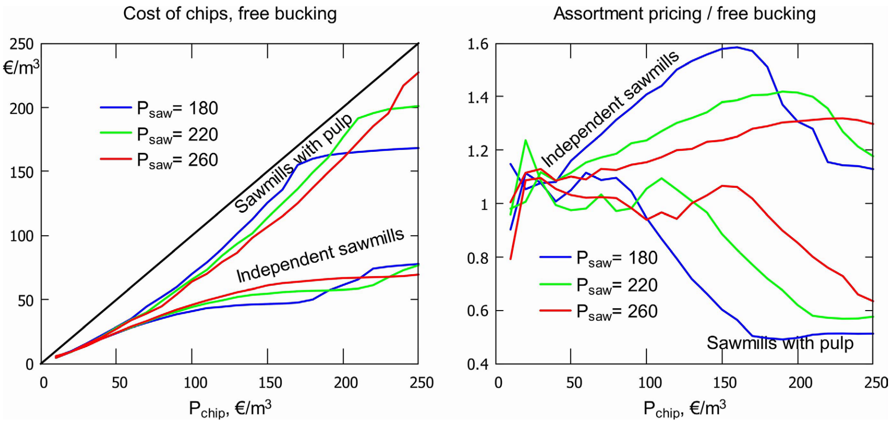

Figure 14. The left subfigure show the cost of additional chips in free bucking computed with Equations (17) and (18) based on maximization of Equations (11) and (12). The right subfigure shows the ratio of costs between assortment pricing and free bucking. The cost of assortment pricing are computed using Equations (19) and (20) based on maximization of Equations (13) and (14).

market. The total supply of chips is limited by the level of potential harvests which can be increased very slowly or not at all. If the company raises the price of chips it pays to get a bigger share of chips at the market, it should evidently pay generally the same price for all chips in the future. But increasing P_{chip} in its own harvests makes it possible to obtain more relative expensive chips without increasing P_{chip} for all chips bought. The company can use its harvests as a similar regulatory reserve for chips as it can use its own forests, or a power company can use waterpower. Figure 14 and Equation (20) shows that the assortment pricing decreases the cost of additional chips: the company can make the forest owner to pay part of the cost of chips by moving chips from the expensive sawlog part into the cheap pulp wood.

5.8. One or Two Sawlogs?

- If sawmills with pulp buck in the assortment pricing one log from stems which could be bucked into two logs, Figure 15 shows that they use in their own harvests much higher price for chips than they pay for independent sawmills. In free bucking, one log is optimal only for very high chip prices.

It is quite easy and painful to forest owners to detect that a buyer bucks one sawlog from stems which would produce two logs. Several forest owners and some previous employees of companies claim that the situation is common in practice.

Figure 15 shows how the share of two-log buckings depends on P_{chip} for three values of P_{saw}. Independent sawmills should always make two logs in free bucking. In the assortment pricing they should buck one log for few stems (left upper corner) if chips have very small value and a stem curve decreases so fast that the second sawlog would produce small sawlog and large volume of expensive chips. Sawmills with pulp should cut one log in free bucking only for very high chip prices. In the assortment pricing, the one-log buckings start to

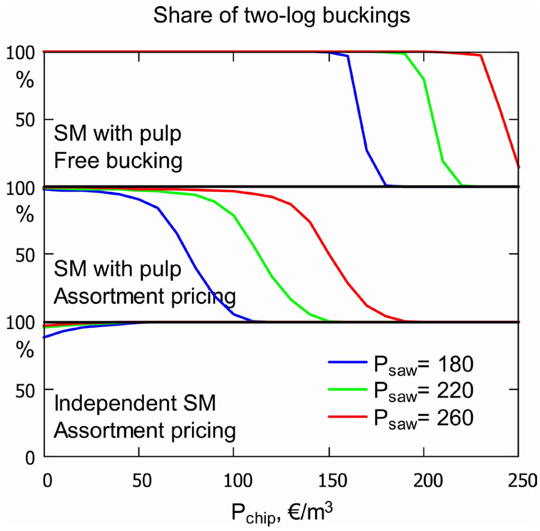

Figure 15. Share of two-log buckings with respect to P_{chip} in the data. Such stems are included where both one and two logs are possible, i.e., $L(D_{\min}) = L(15\ \text{cm}) \geq 2L_{\min} = 80\ \text{dm}$ and making one log with maximum length $L_{\max} = 55\ \text{dm}$ does not allow one further log, i.e., $L(15\ \text{cm}) < L_{\max} + L_{\min} = 95\ \text{dm}$. The are 216 such stem out of 2046.

increase when the chip price starts to increase from 50 €/m³. If the conviction of many people that sawmills with pulp usually buck one log for potential two-log stems is true, this gives evidence that the competition at chip market is not perfect.

It is understandable that a forest owner blames the buyer when seeing how two-log stems produce just one log. The blame has a legal basis only if the buyer has not obeyed agreed log dimensions. Law requires joint-stock companies to behave this way if it is profitable and assortment pricing is used.

6. Sawn Wood Potential Pricing

- Section 6.1 is essential. Section 6.2 shows how the method can consider all the timber up to agreed top diameter. In sawn wood potential pricing, forest owners need to understand Section 6.5. The buyers can utilize Sections 6.3 and 6.4 when moving smoothly to the pricing.

6.1. Definition

The commercial stem volume V_{com} was decomposed above into V_{saw}, V_{chip} and V_{pulp}. If this decomposition is used for pricing, a price should be given to threes component. But pulp wood is also used to make chips for pulping. Longer fibers produced by sawmills are more valuable than fibers from pulp wood. Taking this into account would complicate the pricing. Chips produced by sawmills have larger market value than pulp wood which needs chipping. But we are now considering potential chips in standing trees. V_{chip} contains bark, but so does V_{pulp}.

I suggest the price:

$$\begin{aligned} p_{pot} &= p_{saw}V_{saw}^{\max} + p_{chip}V_{totc} \\ &= p_{saw}V_{saw}^{\max} + p_{chip}\left(V_{com} - V_{saw}^{\max}\right) \end{aligned} \tag{21}$$

where $V_{saw}^{\max}$ is the maximum value of V_{saw} among feasible schedules. V_{com} is obtained from the harvester measurements after a possible update with the top equations shown in **Figure 16**, p_{saw} is the unit price of V_{saw} and p_{chip} is unit price of chips both in V_{pulp} and in V_{chip}. The price can be presented:

$$p_{pot} = \left(p_{chip} + \left(p_{saw} - p_{chip}\right)\frac{V_{saw}^{\max}}{V_{com}}\right)V_{com} \tag{22}$$

Thus

$$p_{com} = p_{chip} + \left(p_{saw} - p_{chip}\right)\frac{V_{saw}^{\max}}{V_{com}} \tag{23}$$

is the unit price of commercial wood. It can be computed both for trees and stands. For stands, $V_{saw}^{\max}$ and V_{com} are sums over all trees and they cannot be computed using stem wise values of $V_{saw}^{\max}/V_{com}$. The analysis that follows aims to justify Equation (23) as a method to calculate the total value of transaction of wood. The purpose is to define the potential volume of sawn wood in a way that captures the essential features of sawing and allows objective comparisons of biddings.

I suggest that the theoretical bucking is done for healthy and undamaged segments of stems. The sawn wood potential price would depend on actual bucking only for finding these segments from the output files of the harvester. It is possible that there would be disagreements between forest owners and buyers about bucking of rotten parts of stems, but these disagreements can be discussed with concrete concepts as the fundamental conflict in the assortment pricing is above rational argumentation.

A buyer needs to evaluate the value of a potential stand considering species composition, logging cost, transportation cost to the mill, branches, sizes of trees and harvest time limitations when setting p_{saw} and p_{saw}. I think that it is important that the price is given to the volume of potential sawn wood and not to value-weighted sawn wood V_{sawu}. This way the amount and value are kept separate. In the trade of raw oil different prices are given to Brent and WTI qualities but amounts are measured similarly. Buyers need well-defined criteria for classifying stands according to their value (Malinen et al., 2015). It is not important for the buyer that it gets a specific stand, it is important that it gets enough stands meeting general requirements. Thus, it should plan a strategy for setting p_{saw} and p_{chip} so that it gets enough stands with minimum total stumpage price and trading costs.

A forest owner needs only to predict $V_{saw}^{\max}/V_{com}$ using stand totals of $V_{saw}^{\max}$ and V_{com} to compare different biddings using Equation (23). The prediction of $V_{saw}^{\max}/V_{com}$ is discussed shortly. I claim that the sawn wood potential pricing fulfills the requirements of the market economy. The method is simple, concrete,

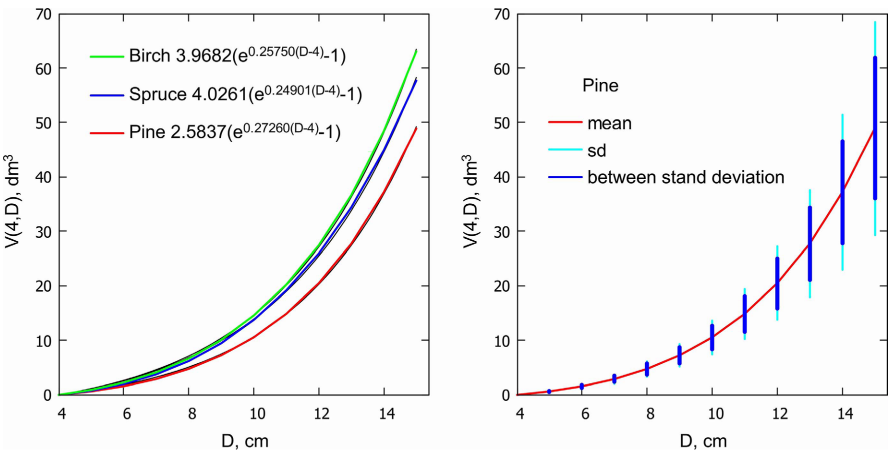

Figure 16. In left figure, colored curves connect averages of $V(4,D)$ from $L(4)$ to $L(D)$ when the x-axis is increased in 1 cm steps. Almost invisible black curves show the estimated curves $ae^{b(D-4)}$ Only such trees are included for which $L(15\text{ cm}) \geq 26$ dm to quarantine that the tops included are not influenced by the butt swelling. On the right, the same mean curve for pine as the left figure augmented with error bars showing the standard deviation (cyan error bars) and the standard deviation of the between-stand effect u in a simple variance component model $y_{ij} = \mu + u_i + e_{ij}$ for tree j in stand i (blue error bars).

transparent, less risky to both forest owners and buyers, decreases trading costs, accounts for tree size and tapering and is easy to compute. For each schedule, V_{saw} can be computed in liters using function $f(D)$ defined in Equation (5) and expressing lengths in decimeters and diameters in centimeters as follows:

```
loglenc=0   ! cumulative length
Vsaw=0
do i=1,nlog   ! nlog = number of sawlogs
loglenc=loglenc+loglen(i)   ! loglen is the log length
Vsaw=Vsaw+f(D(loglenc))*D(loglenc)*D(loglenc)*loglen(i)
end do
Vsaw=π/400*Vsaw
```

I have included pieces of code here and for the bucking schedules earlier to show that the price, and also the results in this analysis, are easy to compute. The sawn wood potential takes sawlog lengths into account only through $L_{\min}$ and $L_{\max}$, where $L_{\max}$ is not important. There is no clear pattern how the price of sawing products depends on the length. In the domestic market, the same price per meter is generally used. Different log lengths around the lengths producing $V_{saw}^{\max}$ provide about the same V_{saw}. So different sawmills with different lengths of sold sawn products can get amount of sawn wood closely related to $V_{saw}^{\max}$ and well predicted from it. Log lengths do not give any clear leverage to forest owners in the pricing.

There are several alternative ways to define the reference bucking which would in practice be about equally good in making biddings comparable. I think that it sounds nice to forest owners that the reference bucking is based on the maximum amount of sawn wood. Buyers should not be worried as they can freely set p_{saw} even if they cannot set $V_{saw}^{\max}$ as they can set V_{log} in the assortment pricing.

Relation of sawn wood pricing to market economy and assortment pricing are discussed in final remarks section.

6.2. Potential Pulp Wood above the Last Pulp Log

- **Figure 16** shows that if a harvester has not harvested all the volume belonging to the agreed commercial volume, the unmeasured part can be predicted well.

If the agreed minimum top diameter $d_{\min}$ is smaller than the top diameter d_m the harvester produces, the theoretical pulp wood should be computed using a model. Reliable models can be made for the volume between diameters d_m and $d_{\min}$, i.e., $V(d_{\min}, d_m)$. In **Figure 16** (left), the black curves describing the theoretical curves are almost invisible under the empirical curves drawn in colors. This is partly an artefact, as the tops are interpolated making the curves more regular than are the true ones. In the used data set, it was possible to derive better predictions using $V(d_m, d_m + 1)$ as a predictor. But harvester data have irregular variation when diameters are close to 4 cm, i.e., when the harvester grip does not have a firm hold of the stem. Thus, such models are not reliable in harvester stem vectors. **Figure 16** (right) shows the means, standard deviations and standard deviations of stand effects in a variance component model when volumes of segments of pine tops were computed at one-centimeter diameter steps starting from 4 cm. Pulp wood volume from d_m to $d_{\min}$ can be computed as $V(4, d_m) - V(4, d_{\min})$. I call this later as 'top correction'.

Volume between the stump and 13 dm, $V(0,13\,\mathrm{dm})$ is predicted in Finland with models
(https://jukuri.luke.fi/handle/10024/554823 based on legislation in
http://www.finlex.fi/fi/viranomaiset/normi/410001/) because harvesters do not provide real measurements close to stump. Smaller grips produce measurements closer to the stump, but it would be too complicate to utilize this. The prediction errors of these model have importance magnitudes larger than errors of the top equations. Prediction errors of $V(0,13\,\mathrm{dm})$ have smaller effect in sawn wood potential pricing than in assortment pricing as in the sawn wood potential pricing errors affect the cheaper chip part and in the assortment pricing expensive sawlog part.

6.3. Bucking Parameters

- Bucking parameters could be selected so that sawlog percentages in the theoretical bucking are the same as current percentages.

A possible criterion to determine the bucking parameters would that they would produce similar V_{log}/V_{com} as the assortment pricing. Those who have access to stem vectors with known bucking results, can use Jlp22 to compute

V_{log} for different parameter sets in the sawn wood potential pricing.

The sawn wood potential pricing makes objective comparison straightforward if different buyers apply the same bucking parameters. As the prices would be under the full control of buyers, no one should have any reason to get the parameters to any specific values. Sawmills sawing small top diameters can show their competitiveness in stands with small trees by increasing p_{chip} as they get sawn wood from that part of stems which is in the reference bucking put to chips part.

Cooperation in the selection of bucking parameters of sawn wood potential pricing would correspond to standardization cooperation of technology industry. The competition authority should not have anything against it. Companies are afraid of accusations of illegal cooperation. Fear of accusations can also be a pretext to avoid market economy in the pricing. Cooperation in bucking parameters could be made openly in front of competition authority, ministry, forest owners and media. A company could start to offer this pricing among other pricing methods. If others would follow, the bucking parameters would evidently reach fixed values. As the new pricing system would need the acceptance of forest owners, their views should be listened, even if the old-time general agreements are no more be possible.

6.4. Setting Prices for Sawn Wood and Chips

- Buyers can estimate prices for sawn wood and chips in the new pricing so that they produce current stumpage prices.

In the assortment pricing, the pulp price p_{pulp} is the price the buyer pays for chips in the pulp wood. The basic idea in the sawn wood potential pricing is that chips in sawlogs have about the same value as chips in the pulp wood. Thus, a good starting value for p_{chip} is current p_{pulp}. As current p_{pulp} may not result from perfect competition, improvement of competition gives pressure to increase p_{chip}. Sawmills use approximately 2.05 - 2.2 m^3 V_{log} for making one m^3 of sawn wood which does not contain saw dust contained in V_{saw}. The saw dust is approximately 10% of V_{log}. Thus $V_{log}\big/\left(V_{saw}-0.1V_{log}\right)=2.05\cdots2.2$, i.e., $0.55V_{log}\le V_{saw}\le 0.59V_{log}$. If the reference bucking produces similar V_{log} as the assortment pricing, it holds that $1.7p_{log}\le p_{saw}\le 1.8p_{1log}$. For the default $p_{log}=75\ €\big/\mathrm{m}^3$ this would mean $127.5\ €\big/\mathrm{m}^3\le p_{saw}\le 135\ €\big/\mathrm{m}^3$.

The buyers need to relate new prices p_{saw} and p_{chip} to the properties of stands. Assume that a company has stem vectors of stands which it has harvested recently. For each stand it can compute $V_{saw}^{\max}$ and V_{com}. Let y denote the stumpage price of a stand, and let x_1 and x_2 be two examples of predictors for p_{saw}, and z_1 and z_2 two possible predictors for p_{chip}. Then, ignoring the error terms:

$$y=p_{saw}V_{saw}^{\max}+p_{chip}V_{totc}$$

$$p_{saw}=a_1x_1+a_2x_2$$

$$p_{chip} = b_1 z_1 + b_2 z_2$$

$$y = (a_1 x_1 + a_2 x_2) V_{saw}^{\max} + (b_1 z_1 + b_2 z_2) V_{totc}$$

$$y = a_1 (x_1 V_{saw}^{\max}) + a_2 (x_2 V_{saw}) + b_1 (z_1 V_{totc}) + b_2 (z_2 V_{totc})$$

Regression parameters a_1 etc. can be estimated with linear regression using $x_1 V_{saw}^{\max}$ etc. as the predictors. The procedure is similar as used in mixed models when random class parameters are predicted with class level variables (Mehtätalo & Lappi, 2020). Statistically determined parameters for expressing p_{saw} and p_{chip} as a function of stand properties is needed only initially when moving smoothly to the new pricing. Thereafter the parameters of functions would be decision variables. When such regression equations are computed there is no need to dig how aggressively the companies have utilized the loopholes of assortment pricing. For winning current biddings p_{chip}/p_{saw} is not important, only the total stumpage price needs to be competitive. But for sending information to the future supply of timber, buyers should tell what they really want.

A company offering stem pricing cannot claim that it would be too difficult to set p_{saw} and p_{chip}. The stem pricing is a special case of sawn wood pricing setting that $p_{saw} = p_{chip}$.

After a smooth transition to sawn wood pricing, Adam Smith's invisible hand can take control of p_{saw} and p_{chip}.

6.5. Estimation of Sawn Wood Percentage

- A forest owner needs in the new pricing an estimate for the sawn wood percentage to compare biddings. This can be estimated objectively.

In the sawn wood potential pricing, an estimate for $V_{saw}^{\max}/V_{com}$ needs to be computed to get the unit price of commercial volume using Equation (23). If $V_{saw}^{\max}$ and V_{com} have a permanent definition and do not depend on the realized bucking, it is possible to accumulate knowledge for their estimation. In the sawn wood potential pricing, the effective total stumpage price is unknown at the time deal, but it can be estimated objectively. This is at contrast to the assortment pricing where V_{log}/V_{com} is secret and unpredictable. **Figure 17** (left) shows how regularly $V_{saw}^{\max}$ and V_{com} depend on *dbh*. It would be easy to estimate reliable models for these curves. The most difficult task is to estimate the diameter distribution cost-efficiently. In estimating $V_{saw}^{\max}$ and V_{com} it should be considered that $V_{saw}^{\max}$ and V_{com} are nonlinear with respect to *dbh*. Thus values of $V_{saw}^{\max}$ and V_{com} of curves at average *dbh* give biased (too large), prediction for averages of $V_{saw}^{\max}$ and V_{com}. If in addition to mean also standard deviation is estimated, the bias can be corrected using methods presented in Mehtätalo and Lappi (2020). The two-point distribution method is my favorite. Instead of using the arithmetic average of *dbh*, e.g. the basal are median *dbh* may be more useful. Tree wise $V_{saw}^{\max}/V_{com}$ is not useful for estimated the stand level ratio of $V_{saw}^{\max}$ and V_{com}. In the whole data the average of tree wise $V_{saw}^{\max}/V_{com}$ was 28.8%, and total $V_{saw}^{\max}$ was 47.5% of total V_{com}. It is possible that reliable estimates of

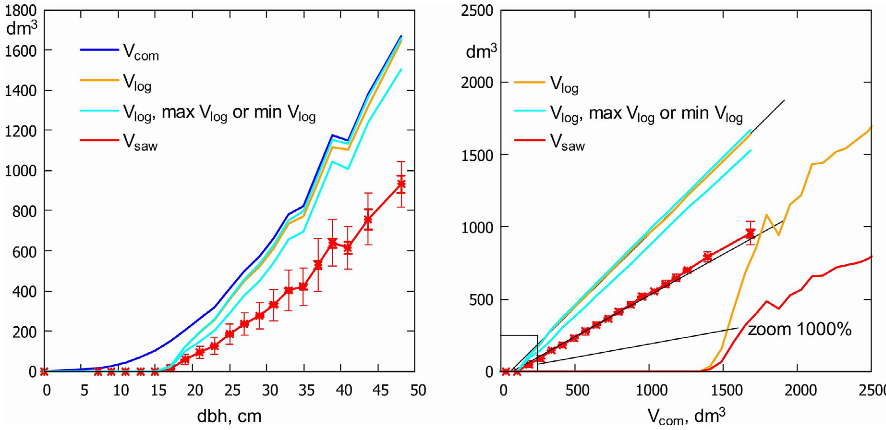

Figure 17. Left figure shows how commercial volume V_{com}, sawlog volume V_{log} (orange line) and sawn wood volume V_{saw} depend on *dbh* when V_{saw} is maximized. Note that V_{saw} is in fact equal to $V_{saw}^{\max}$. Cyan lines show the range of possible sawlog volumes. Right figure shows how V_{log} and V_{saw} depend on V_{com}. Black lines are the regression lines. When all variables were multiplied with 10, the orange and red line on the bottom right corner are obtained, the ranges of axes becoming (0, 250) instead of (0, 2500).

$V_{saw}^{\max}/V_{com}$ can be obtained directly instead to estimate first stand wise $V_{saw}^{\max}$ and V_{com}.

Figure 17 (right) shows the stem level dependency of V_{log} and $V_{saw}^{\max}$ on V_{com}. The overall figure is dominated by large trees and indicates precise linear relation: R^2 was 0.99 in linear regression for V_{log} and 0.98 for $V_{saw}^{\max}$. Zooming on trees having $V_{com} \le 200\ \text{dm}^3$ shows that small trees behave differently. Recall that V_{log} is not used in the sawn wood potential pricing.

7. Comparing Biddings

- The unit price of commercial wood can be used to compare biddings. In the assortment pricing, the unit price is secret during the deal.

Two biddings using sawn wood potential pricing with same bucking parameters can be compared using Equation (23) after estimating $V_{saw}^{\max}/V_{com}$ either directly or estimating first $V_{saw}^{\max}$ and V_{com}. Two biddings with different bucking parameters can be compared using Equation (8) (or its tuned version) to make values of $V_{saw}^{\max}/V_{com}$ comparable. In stem pricing, a buyer sets the same unit price for all commercial volume obtained in the harvest. The unit price can be compared to p_{com} after the top correction shown in **Figure 16**.

M-Group offers size-dependent stem pricing where the unit price of commercial volume depends on the tree-level average of commercial wood. After estimating the average stem size (the buyer provides an estimate) for size-dependent stem pricing and $V_{saw}^{\max}/V_{com}$ for sawn wood pricing, a forest owner can compare the implied unit prices of commercial wood after the top correction. I discuss stem pricing later.

In the assortment pricing, the unit price of commercial wood can be presented similarly as in the sawn would potential pricing as

$$p_{com\_a} = p_{pulp} + \left(p_{log} - p_{pulp}\right)\frac{V_{log}}{V_{com}} \quad (24)$$

Because V_{log}/V_{com} is a business secret of the buyer, the forest owner cannot compare sawn wood potential pricing objectively to Equation (24) even if the owner would know each stem precisely. Forest owners selling often timber, or forest owner associations can collect information of V_{log}/V_{com}. But it is difficult to get consistent results because the price cycles of sawn wood and pulp and the chip needs for even flow of chips vary over time leading to irregular V_{log}/V_{com}. Despite of these problems, a forest owner should concentrate on Equation (24) instead of paying attention to p_{log} and p_{pulp}. Sawlog price p_{log} may psychologically look more important than it is, especially if high p_{log} is connected to low V_{log}. If a forest owner sells timber using assortment pricing and there were biddings using stem pricing or sawn wood potential, the owner can know after the deal whether the correct selection was made.

8. Discussion

8.1. Interpretation of the Results

Independent sawmills may consider that both in the assortment pricing and in the sawn wood potential pricing higher chip prices make longer logs more profitable (**Figure 11**).

The analysis of the assortment pricing assumed that the buyer has already bought the stand using a given p_{log} and p_{pulp}. But buyers need to consider bucking and determination of p_{log} and p_{pulp} simultaneously so that it wins enough biddings. This is demanding if a sawmill with pulp plans to buck using high P_{chip} leading to reduced V_{log} and reduced stumpage price.

I made first round of computations using sawn wood volume V_{saw} in the objectives instead of the value-weighted sawn wood volume V_{sawu}. Result looked otherwise similar as above, but the drop of curves was even faster than in **Figure 10** when price of chips P_{chip} approaches price of sawn wood P_{saw}. Optimization of V_{saw} leads to on/off behavior while optimization of V_{sawu} leads to smoother increase of chips.

Common view of many forest owners and experts is that sawmills with pulp produce more unpredictable and more variable bucking results than independent sawmills. **Figures 8-10** show that the validity of this claim is dependent on the value of P_{chip} companies use in the bucking. If it would be possible to access empirical data of bucking results it might be possible to estimate P_{chip} they use in the bucking. Then correction coefficients could then be estimated to make the saw-log prices of sawmills with pulp comparable to the sawlog prices of independent sawmills. If such correction coefficients would be published, companies owning both pulp mills and sawmills might be more willing to accept sawn wood potential pricing in the timber trade. Some forest owner associations collect such information but are of course not willing to publish it.

Above analysis makes it understandable why companies have kept secret their price matrices. When looking more closely to the bucking, I was surprised how simple is the maximization of the value of sawn products in the bucking. As all sawmills operate at the same sawn wood market and harvest similar stands, the competitors may not benefit of knowing what log dimensions are made. The only real secret is the regulation of the sawlog percentage with the secret price matrix.

If the companies change prices, the profit maximization requires the companies to change bucking in the assortment pricing simultaneously. I did some computations with alternative elasticities. The results might be of some interest to economists who have wondered peculiar behavior of elasticities in the lumber trade, but as they do not bring a new feature into the whole picture, I'm not showing them. This may be one reason, why estimating price elasticities for supply and demand is so difficult, and the results vary much from study to study (Kallio, 2001; Tian et al., 2017). The assortment pricing would require economic theories for the supply and demand when prices are secret and selection of the buyer is a lottery game. If buyers change sawlog and pulp prices and bucking simultaneously, and the forest owners sell whole stems, and the forest owners cannot objectively compare effective total stumpage prices, it would be a scientific miracle, if consistent results would be obtained for elasticities. In the sawn wood potential pricing, prices are given for such quantities of which the buyers are interested. The prices are not nested as p_{pulp} is nested in p_{log}. If the sawn wood potential pricing will be used, I anticipate more consistent elasticities.

The analysis was based on stem components and their prices. A harvester bucks using a price matrix for combinations of lengths and top diameters of sawlogs and pulp wood. If V_{sawu} is used in the objective, sawlog value $P_{saw}V_{sawu}$ can be directly put into the price matrix because V_{sawu} depends only on the log length and top diameter. The component V_{chip} can be put into the price matrix by writing $V_{chip} = V_{log} - V_{saw}$. The harvester can predict V_{log} using the stem curve model it uses in its bucking algorithm.

Capacities of sawmills were not considered above in the analysis. We may assume that independent sawmills do not buy stands whose sawlogs it cannot saw. Sawmills with pulp need to consider both the capacity of the sawmill and the capacities of pulp mills of the company. Sawmill and pulp mill capacities affect differently. Sawmill capacity sets an upper limit for the production. A pulp mill capacity sets both upper and lower limit for the flow of timber. This emphasizes again the role of P_{chip} in the bucking. Currently companies sell and buy timber fluently from each other. Thus, a sawmill with pulp can sell sawlogs which exceed the sawing capacity, and it can send them also to pulp mills.

Results above were computed using $p_{log} = 75\ €/\mathrm{m}^3$ and $p_{pulp} = 25\ €/\mathrm{m}^3$. Increasing p_{log} has the same effect as decreasing both P_{sawu} and P_{chip}. Increasing p_{pulp} has the same effect for sawmills with pulp as decreasing P_{chip}.

There are four different reasons to 'transport valuable sawlog to pulp mill' in the assortment pricing, i.e., to buck smaller V_{log} than possible. These reasons for sawmills with pulp are, starting from $P_{chip} = 0$ (**Figures 4-6**).

Both the amount of sawn wood V_{saw} and its value V_{sawu} can be increased if their maximum is on the left from the cutting point producing maximum V_{log}.

Moving left from the cutting point maximizing V_{saw} decreases both V_{log} and V_{saw} but increases the value of sawn wood V_{sawu} because the top diameters D_{top} increase.

Moving left from cutting point maximizing V_{sawu} decreases V_{log}, V_{saw} and V_{sawu} but decrease of V_{sawu} is smaller than the benefit of moving expensive chips from the sawlog to cheap pulp wood. Dead weight loss is large (**Figure 13**).

When chips are very valuable, then bucking even less V_{log} means that also valuable sawn wood in the sawlog is moved to valuable chips in the pulp wood V_{pulp}. The forest owner gets a big loss compared to bucking maximizing V_{log}, V_{saw} or V_{sawu}, and comparing what an independent sawmill had paid with clearly smaller nominal sawlog price p_{log}. Dead weight loss is, however, small (**Figure 13**).

The above arguments apply directly when considering bucking of the last sawlog. With several logs the same reasoning applies in principle, even if there will be more complicated interactions.

8.2. Stem Pricing and Other Pricing Methods

Hekkala (2023) describes properties of stem pricing and other pricing methods. Stem pricing was already analyzed above from the viewpoint of bidding comparisons. Now its utility is discussed for the viewpoint of market economy, trading costs and optimization of silviculture. In stem pricing, the buyers need to spend more money than in the sawn wood potential pricing to study the properties of the stand to make a reasonable bidding. The forest owner cannot see how the offered price is related to the value of wood in wood processing. The pricing does not give any information for forest owners and research how to optimize future forestry or how to select the next stand to harvest.

Size-dependent stem pricing gives a little information of what kind of models are behind the price. But as a confidential price is given for each forest owner and stand separately for a narrow stem size window in tabular form, no general understanding is obtained. The only table I have seen indicates that the dependency of price on the stem size consists of two linear parts having almost identical slopes. I cannot see any other reason for presenting price in tabular form instead simple function than that M-group wants to hide from supply side of timber trade the demand it has for timber.

In stem section pricing the different stem sections defined with diameter ranges are given different prices. This pricing will evidently become important when sawmills start transporting whole stems into sawmills and start to saw stem sections. I do not see any utility in this pricing if sawmills sawlogs. I'm not aware of any quantitative analysis of how stem sections are related to sawn wood and chips.

Matrix pricing defines a separate price for each diameter, quality, and length class. It would be impossible for the forest owner to compare price matrices of different buyers and to understand their background. Only companies use matrix pricing in selling sawlogs to other companies (Hekkala, 2023).

8.3. Distribution Bucking

In distribution bucking trees are bucked so that the distribution of log lengths and top diameters is close to a target distribution (Kivinen, 2001). I think that this approach looks the bucking problem from a false direction. This approach would be rational if there are more trees available than needed and it would be possible to just pick from the large population the distribution wanted. When the harvester grip moves on $D(L)$ in **Figure 3**, **Figure 5**, and **Figure 15**, only such dimensions can be made which are on the stem curves. A user of the distribution bucking is like a passenger of a train wanting to go to a town outside the track. There are studies of how closeness to the target distribution should be measured (e.g. Malinen & Palander, 2004). If the top diameter is larger than a saw blade setting requires it can still be sawn with the same setting as a smaller top diameter cannot. One saw blade setting may allow d_{top}-range of 0.5 - 2 cm (Pertti Holmila and Antti Heikkilä, personal communication). In bucking the difference between d_{top} and D_{top} needs to be considered. It is possible, even if not rational, to find bucking schedules producing a distribution close to the target using Jlp22 with a goal programming formulation (Lappi, 1992).

The target distribution of sawlog dimensions may be sufficient for providing the target distribution of sawn product dimensions. But a specific target distribution of log dimensions is never necessary. It is likely that a target distribution of logs is not feasible, but there are feasible distributions that produce desired sawn products. In the further research section, I'll outline how linear programming can be used to get desired sawn product distribution both when there is one evident saw blade setting for each top diameter or there are several settings for some top diameters.

The message of this comment is the same as the message of the sawn wood potential pricing: pay attention to sawn wood and sawn products, not to sawlogs. Researchers could better help sawmills if sawmills would describe their bucking problems in terms of blade settings and dimensions of the products instead of target distributions of logs. It is a common fallacy in human thinking to lose sight of end goals while optimizing means goals.

8.4. Rotation

Rotations implied with fixed prices p_{saw} and p_{chip} are longer than rotations computed assuming corresponding fixed prices p_{log} and p_{pulp} because $V_{saw}^{\max}/V_{log}$ and thus price for V_{log} increases when trees become larger (**Figure 8**). Fixed p_{saw} would not consider all benefits of larger logs. So, in a genuine market economy p_{saw} would be larger for stands having larger trees and this should be considered in rotation studies.

8.5. Bucking Simulator for Harvesters

The assortment pricing does not encourage attention to the bucking algorithm of the harvester. For the harvest contractor, the speed of harvesting is the most important thing. Buyers do not care of poor bucking because forest owners pay for it, and bucking producing small V_{log} may be just what they want. Forest owners might be interested, but they do not select the contractor and they do not get the stem vectors so that they or their consultants could study the bucking results.

If price is independent of bucking, it would be in the interests of buyers that harvesters buck well. They cannot buck perfectly because the harvester does not know the stem above the grip. It is difficult to make empirical studies for comparing different harvesters. It would be simple to make a simulated competition between algorithms. A neutral organization would send to the competitors stem measurements decimeter after decimeter similarly as the harvester grip sends them to the computer of the harvester. The competitors should send back commands: break, slow, stop, cut, withdraw, full speed. Researchers and other firms could participate in the competition. When the results of the competition would be published, this would give incentive to harvester manufacturer to develop the bucking algorithm or buy a better one. Currently harvester manufacturers have no incentive to develop bucking algorithms.

A reasonable bucking algorithm of a harvester uses a statistical model which predicts both diameters above the harvester grip and provides estimates both the standard deviations and correlations of the prediction errors. The stem curve models in Lappi (1986) and Varjo et al. (2006) provide such estimates. Underprediction of the diameter at a planned cutting height is less harmful than overprediction. Thus, sequential stochastic optimization is a decent methodological framework for the optimization (De Pellegrin-Llorente, 2022).

8.6. Database for Stem Vectors of Harvesters

During the harvester decades, a huge amount of valuable information has been lost as researchers have not had access to the data. An evident explanation is that the companies have been afraid of showing the stem vectors to forest owners who could compare the bucking results to the bucking possibilities. If pricing is independent of bucking, the companies have no reason to hide the stem vectors. They can freely hide their bucking results.

Companies owning both sawmills and pulp tell that they own the stem vectors and forest owners do not have right to access them. Independent sawmills might be more willing to give researchers access to stem vectors. If sawmills with pulp own the stem vectors, independent sawmill own evidently also, even if they may not be aware of it.

I suggest that a general data base for stem vectors should be established to which harvesters could initially send, after removing the identity of the forest owner, sample stem vectors. Initially only voluntary owners of the vectors could be used. After needed legislation, this could be obligatory. Luke would be a natural organization to take care of the data base together the inventory data, but only if it provides free access to data. If such data had been collected from the beginning of harvester epoch, good thinning models could now be estimated from stands harvested for a second or third time. Benefits of harvester data are discussed by Kemmerer and Labelle (2021).

Finnish researchers of medicine have had the advantage that they have better access to medical data bases than researchers in most countries. They have had better access to syphilis data of people than forest scientists have had to stem vectors. I was not able to access harvester data for this paper. With harvester data from several stands, I could have studied the between-stand variation of the studied variables.

8.7. Further Research

If a sawmill has estimated the diameter distribution for a stand to be harvested and can predict the stem curve for each *dbh,* or there are data from a similar stand, Jlp22 can be used to generate such price matrix that desired amounts of sawn wood products or chips are obtained. First, *stemopt* function is asked to write all bucking schedules for each *dbh* to disk. Then such linear programming problem can be described with *problem* function which produces desired amounts of products. Then, *jlp* function can be used to solve the problem.

There are two different ways to formulate the problem. If for each top diameter of log there is only one evident saw blade setting, each log-length and top diameter combination is augmented in the bucking schedule data with the amounts of product. Then the LP problem and data have the same mathematical structure as the traditional forest management planning problems for which the linear programming algorithm of Jlp22 is designed. Trees having a given *dbh* correspond to a stand in management planning problem. The problem formulation is easy and the solution time short. When the problem is formulated using dummy variables for sawlog length and top diameter combinations, the shadow prices of the solution give the desired prices for log length, top diameter combinations.

When the shadow prices are solved using a predicted diameter distribution, it may become apparent during the harvest that desired sawn wood dimensions will not be obtained. Then the shadow prices need to be updated either resolving the LP problem with the updated diameter distribution or using the heuristic updating of Dtran algorithm (see De Pellegrin-Llorente, 2022). When moving to forest level, the study of Laroze (1999) should be consulted.

If several blade settings are allowed for some top diameters, the problem has the same structure as planning problem where timber is transported to factories (Lappi & Lempinen, 2014). A blade setting is then a 'factory' to which logs are transported. A simpler and more efficient algorithm for factory optimization is under implementation to Jlp22. I would try to prove my criticism of distribution bucking in section 8.3 if I could access harvester data.

A sawmill can optimize blade settings with Jlp22 when the sawlog storage, blade settings and sawn wood sales and/or sawn product prices are given. This problem has also the same mathematical structure as the traditional management planning problems. The problem formulation requires that the log storage, blade settings and product sales data sets are linked to each other in a convenient way. New tools for that will be published soon. The method can be developed and published without real data.

I try to finalize my study for the prediction of stem curves above the harvester grip and for implied stochastic optimization of bucking. I do not mind if someone makes it first.

8.8. Final Remarks

8.8.1. Assortment Pricing

The dead weight loss is not the most important damage to the society what the assortment pricing does. More important costs of the assortment pricing are:

- It prevents Adam Smith's invisible hand to lead timber to buyers who can most efficiently utilize it.
- Because the relation between nominal sawlog and pulp prices to the effective stumpage price is arbitrary, timber prices do not give information for future harvests. It leads harvests to stands which are not most appropriate to the current phase of pulp and sawn wood price cycles and to nonoptimal rotations.
- Bucking foreseeing is big business to forest owner associations. In free bucking, the cost of foreseeing sawlog percentage and fighting against foreseers could be saved. The Central Union of Agricultural Producers and Forest Owners participates in bucking disputes. It may be willing to get rid of the costs of the disputes even if the disputes demonstrate to forest owners that it fights for the forest owners.
- A forest owner association may now classify a sawmill with pulp as buyer with small sawlog percentage. If a sawmill would be willing to buck more V_{log} due increased value of sawn wood, it may not be able demonstrate its changed bucking policy if it does not win biddings.
- As the increase of value with respect to stem size is not considered, buyers need to give different sawlog prices p_{log} for stands having different sizes. Buyers need to spend money to predict the size distribution before making a reasonable bidding. As the buyers do not explicitly describe this to forest owners, many forest owners getting smaller prices think that they are not treated fairly. Some buyers try to hide that they pay more for large stems by giving free services in the regeneration. The sawn wood potential pricing takes the dependency of value on size automatically into account. There would be less variation in p_{saw} than there is in p_{log}.
- In the assortment pricing, buyers have objective reasons to treat different owner groups differently. Small sawlog percentages need to be bucked with as small damage to the reputation as possible. Companies need different

strategy with respect to forest owners selling regularly, and for forest owners living in the countryside and receiving information from neighbors and knowing more of forestry. Buyers often give estimates for the total stumpage price. It is useful to overestimate the stumpage price for such forest owners who will not spread information of overestimations. If a forest owner association helps the forest owner in the trade, the buyer cannot freely reduce V_{log}. There are rumors that if an employee of the association foresees bucking too actively, sawmills with pulp do not make biddings if he intermediates them to forest owners.

- Agreed dimensions are problematic for a buyer if there is time lag between the deal and the harvest and the market of sawn products has changed (Hekkala, 2023).
- The inevitable conflict of interest between forest owners and companies causes mistrust whose effect is difficult to measure. Forest owners feeling that they have not treated fairly may stop selling timber altogether.
- Bucking disputes waist money and damage the fame of the whole forest sector. If a company wins a dispute, it validates the common belief that legal system obeys companies. If a forest owner wins a dispute, it gives evidence that companies cheat forest owners. Pricing preventing the buyer to handle the bought commodity in desired way is simply absurd. Assortment pricing is a shame for the forest sector.
- Companies need to spend money to advertise their reliability and emphasize trust as there is no objective ground for the trust.
- Assortment pricing leads to hiding of valuable harvester data from researchers.
- Economic theories of the relations between price, supply and demand assume that the supply-side knows the price. This does not hold in the assortment pricing where the effective price is a business secret during the deal.
- In market economy, the profit maximization of different participants is beneficial to the whole economy. Without profitable forest industry there is no profitable forestry. Without profitable forestry, the industry does not get raw material. When considering a concrete timber deal in the assortment pricing, a buyer does not think that a competitor with eventually better bidding could perhaps more efficiently utilize the harvested stems, or that bucking stems so that the forest owner loses income makes good silviculture less profitable. Full benefit of sawn wood potential pricing is obtained only if buyers openly tell their pricing policy instead of giving confidential biddings for each forest owner and stand separately as is currently done.

Historians may explain why assortment pricing has lasted decades after harvesters made it irrational from the viewpoint of market economy. An evident explanation is path dependency. Another explanation is that market economy is not beneficial, at least in short run, to all partners of timber trade. Specifically, it is beneficial to sawmills with pulp that forest owners cannot objectively compare biddings. They can win biddings with high sawlog price and decrease stumpage price with small V_{log}. It is beneficial to them also that they can keep oligopoly and pay different price for chips they obtain from independent sawmills than they use in their own buckings. Timber trade is a triangle drama between forest owners, independent sawmills and companies owning both sawmills and pulp mills. Even if many people see the problems in the assortment pricing, people are reluctant to shake the system which is obtained a balance of fear.

Economic growth has always been linked to the growth of trade. The growth of trade has been always linked to more strict regulation of the rules of trade: buyers must be able to trust that they will get what they have bought without seeing the commodity, and sellers must be able to trust that they will get money for what they have sold. An essential part of the regulation are the rules how quantities are measured. In timber harvest, there are in Finland strict rules for the calibration of the harvester measurements and for the estimation of the stem volume up to 13 dm.

In 1740 in Sweden grain was sold in kappas, but kappas of different sellers had different sizes. Most clever sellers had secret double bottoms. In 1743, king Frederick I legislated legal dimensions of kappas and their inspection and stamping. In the assortment pricing, the situation is like in the grain trade before the 1743 law: different buyers measure different sawlog volumes from the same stems. King Frederick I understood better in 1743, decades before Adam Smith's Wealth of Nations (1776), the requirements of fair trade than forestry sector of Finland A.D 2024.

8.8.2. Benefits of Sawn Wood Potential Pricing

- Sawn wood potential pricing combines best aspects of assortment pricing and stem pricing: forest owner can objectively compare biddings and forest owner can understand how p_{saw} and p_{chip} are related to the products obtained from stems. In the assortment pricing p_{log} and p_{pulp} give the same this information but in misleading form as p_{log} applies both the sawn wood and chips within the sawlogs. Kallio (2001) describes the dependency between p_{log} and p_{pulp} 'the pulpwood price should be nested in the sawlog price'.
- Currently published statistics of sawlog price p_{log} and p_{pulp} give some information about demand cycles of sawn wood and pulp. Because only a part of sawlog volume goes to sawn wood and the relation between nominal prices and effective stumpage price is weird, this information is obscured. Statistics of p_{saw} and p_{chip} in the sawn wood potential pricing would put this information to a higher level.
- A forest owner needs not to know what criteria buyers have used in setting p_{saw} and p_{chip} for a stand when selecting best bidding. But if buyers tell even qualitatively their criteria, forest owners and research can take these criteria into account in future silviculture.
- Forest owners would evidently react better to the changes of prices than in the assortment pricing where price and amount are intermingled. Smaller

price changes would be needed to inform of changes of the demand.

- From buyer's point of view, sawn wood potential pricing is less risky than stem pricing for two reason
    - As sawn wood potential pricing takes the utility of larger and slowly tapering stems into account, buyers need not to predict the size distribution and tapering of stems so well as in the stem pricing. Sawn wood potential pricing is less risky for the buyer, and it is cheaper to make a bidding. The size-dependent stem pricing of M-Group takes the average size into account, but as the value is nonlinear with respect to size, average size is not enough to predict the value of the stand.
    - The sawn wood potential pricing makes theoretical bucking into healthy and undamaged parts of stems. Thus, the potential damages and diseases of stems are not risky to the buyer. Forest owners may be more willing to accept that they do not get full price of rotten stems than the fact that buyers need to give lower price to all forest owners to cover with a sufficient margin the risk of rotten stems.
- Many independent sawmills are specialized to utilize efficiently stems of special size or type. Some buyers buck also small logs having different dimension requirements than sawlogs. They can show their competitiveness in the sawn wood potential pricing contrary to the assortment pricing.
- For sawmills with pulp, optimization of bucking is straightforward in free bucking as they need not consider whether chips come from sawlogs or pulp wood. The relation between sawn wood and chips can be solved purely by price cycles of pulp and sawn wood.
- From the buyer's point of view, it is reasonable to vary $d_{\min}$ according to price cycles of pulp or to put more wood to tree tops used for energy, In the sawn wood potential pricing, buyers can use their own $d_{\min}$ in bucking and standard $d_{\min}$ in the bidding. In the assortment pricing, differences between realized and agreed $d_{\min}$ cause additional mistrust. Variation of $d_{\min}$ in the sales contract would confuse forest owners further.
- Sawn wood potential pricing would probably lead to more perfect competition at the chip market.
- Forest owner associations could concentrate on foreseeing that thinnings are done properly and remaining trees are not damaged. Their help would be needed to estimate $V_{saw}^{\max}/V_{com}$. The sawn wood potential pricing would remove only their irrational tasks.
- Currently inventory and allowable cut results are published in terms of sawlog and pulp volumes. It would be more informative to use sawn wood and chip volumes.
- Developing growth and yield models for sawn wood potential and volume of chips would give better basis for optimization of management than models for sawlog volumes and total volumes because the thickness and tapering and their interaction would be considered.
- The stems are bucked into sawlogs and pulpwood for obtaining chips and

sawn wood. The better p_{saw} and p_{chip} correspond to P_{saw} and P_{chip} used in the bucking algorithm, the better Adam Smith's invisible hand can work.

- The sawn wood potential pricing would allow fair and hard competition. Timber trade game would be like chess. The competition can be hard also in the assortment pricing, but the game is like poker where bluffing is so important.
- In sawn wood potential pricing buyers winning biddings in the assortment pricing with smaller effective stumpage price and forest owner associations overseeing bucking would lose. There is no value-free way to compare these losses to the above benefits.

8.8.3. Moving to Sawn Wood Potential Pricing

For the sawmills with pulp, the most critical question in the comparison of pricing methods is the role of P_{chip} in their bucking algorithm, and generally in their chip strategy. If it is essential to prevent perfect competition at chip market and to have possibility to buck small log volumes for selected forest owners, they may be reluctant to accept the market economy.

Stem pricing has opened the road to market pricing. Sawn wood potential pricing develops stem pricing further. As Equation (23) provides just one unit price for the total commercial volume in a stand, buyers using stem pricing can define infinite number of equivalent biddings in the sawn wood potential pricing. But if the buyers do not tell their true p_{saw} and p_{chip}, the prices would not give information for the optimization of future supply.

The popularity of stem pricing is increasing. If timber trade goes from assortment pricing to stem pricing, and then later from stem pricing to sawn wood potential pricing, this would be an example of Hegel's dialectic development from thesis to antithesis, and from antithesis to synthesis.

If forest owners could get access to the stem vectors it would be easy to compute with Jlp22 what kind of bucking results had been possible among agreed log dimensions. It would also be possible to estimate what P_{chip} has used in the bucking. With sufficient data it might be possible to estimate correction coefficients for nominal prices. Some forest owner associations already collect statistics of sawlog percentages of different buyers, but they do not publish them. I'm convinced that if forest owners could access stem vectors, they would not tolerate the assortment pricing very long.

EU has much legislation to protect consumers, buyers. For instance, Consumer Protection from Unfair Trading Regulations, 2008 'Prohibit traders across all sectors from using unfair commercial practices that hinder consumers from making informed purchasing'. In timber trade, sellers should be protected from practices that hinder them making informed sales. Brignull (2023) analyzes tricks companies use to deceive consumers. Finnish politicians claim that EU interferes too much in the forestry matters of Finland. The fact that Finland has been stuck for decades to the corruptive assortment pricing conflicting with market economy and principles of fair-trade shows that Finland cannot take care of the timber trade in a rational way without the overseeing of EU. Finland may understand better its forest than EU bureaucrats.

Forest owners are used to assortment pricing as their fathers and grandfathers were. An evident doubt to the sawn wood potential pricing is that forest owners could not understand it. According to my experience forest owners can understand many basic things better than the forest experts. I'm sure that forest owners would understand sawn wood pricing when seeing **Figures 1-4** with proper explanations. Toivo Hyvärinen, forest owner who won at the court of appeal a bucking dispute in assortment pricing with UPM, understood well my pricing suggestion and thought that with such pricing similar disputes would not rise (personal communication).

A problem in any pricing is that most forest owners expect to get at least the average price. This is one reason why buyers pay under the table for valuable stands to get smaller prices into the official price statistics. As the sawn wood potential pricing takes automatically into account a significant part of the increased value of larger trees, p_{saw} would vary less than effective p_{log} in the assortment pricing. If the companies would openly tell what criteria they are using in the determination of the price, it would be easier to a forest owner to believe she/he is not treated unfairly if she/he gets less than a neighbor.

Sawn wood potential pricing would give a solid base for long term development of forestry and forest industries. Forest sector faces big challenges due to the climate change, environmental requirements, and EU regulations. If the absurd conflict of interest between forest owners and forest industry in the assortment pricing will be removed, forest sector could more united face the future challenges.

8.8.4. Suggestions

Following suggestions are based partly on my personal views of fair play, partly on the above analysis. They are not value-free.

- Forest owners could finally understand that the buyers are not interested in the sawlog volumes but in sawn wood and chips. They should not take the sawlog prices of sawmills with pulp seriously in the assortment pricing.
- Ministry could make legislation for
    - permitting forest owners to access stem vectors. I could then advice how they could compute with Jlp22 what kind sawlog volumes had been possible.
    - establishing data bank for harvester vectors.
    - making statistics for p_{saw} and p_{chip} if sawn wood potential pricing will be accepted.
- EU could
    - modify laws protecting consumers to protect sellers when buyers are multibillion companies and sellers are ordinary people.
    - study whether it has means to force Finnish timber trade to fair trade and market economy.

- Independent sawmills could learn to ask harvest contractors to send hpr files containing stem vectors. They could then study bucking optimization or ask researchers to do that.
- Timber buyers could start to compare bucking results to bucking possibilities using Jlp22 to better evaluate contractors and harvesters.
- Companies owning pulp mills could make a leap to market economy in the spirit of the leap of faith of Søren Kierkegaard. Independent sawmills would easily follow.

Acknowledgements

Heikki Smolander raised my interest in the topic criticizing for years assortment pricing saying that if similar pricing would be used for slaughtered animals, the farmer would get different prices for meat pieces cut to round or mince, and special foreseers would control in slaughterhouses that a piece which could be used for round is not used for mince. Tommy Lindström, chief executive of 100-year-old Iisveden Metsä sawmill gave valuable information for this paper and for my forthcoming book from the perspective of independent sawmills. Antti Heikkilä provided **Figure 1** and data for **Figure 2**. He and Pertti Holmila taught me many aspects of sawing. Erno Vesterinen, a local sawn wood seller from Rakennuspari, told how domestic sawn wood trade works. Toivo Hyvärinen, forest owner who won a bucking dispute with UPM told his experiences of sawmills with pulp, and he proved that forest owners may easily understand my pricing suggestion. Jari Kuuluvainen made useful suggestions for an earlier version of the paper. I had useful discussions of my paper with Lauri Mehtätalo and Reetta Lempinen from Luke, Erno Järvinen from the ministry of agriculture and forestry, Mikko Tiirola and Pauli Rintala from The Central Union of Agricultural Producers and Forest Owners (MTK), Paula Horne from Pellervo Economic Research, Anu Islander from Forest Industries and Eeva Korpilahti. Pekka Kauppi suggested to make a paper in English instead of a discussion paper in Finnish. Risto Päivinen suggested to change the title to more appealing. Reetta Lempinen allowed me to see timber trade biddings she has obtained.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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